

USM-TH-209

## Further results for the two-loop $L_{cc}$ vertex in the Landau gauge

Gorazd Cvetič<sup>a</sup> and Igor Kondrashuk<sup>a,b</sup>

(a) *Centro de Estudios Subatómicos y Departamento de Física,  
Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile*

(b) *Departamento de Ciencias Básicas,  
Universidad del Bío-Bío, Campus Fernando May, Casilla 447, Chillán, Chile*

### Abstract

In the previous paper hep-th/0604112 we calculated the first of the five two-loop diagrams for the  $L_{cc}$  vertex of the general non-Abelian Yang-Mills theory, the vertex which allows us in principle to obtain all other vertices via the Slavnov-Taylor identity. The integrand of this first diagram has a simple Lorentz structure. In this letter we present the result for the second diagram, whose integrand has a complicated Lorentz structure. The calculation is performed in the  $D$ -dimensional Euclidean position space. We initially perform one of the two integrations in the position space and then reduce the Lorentz structure to  $D$ -dimensional scalar single integrals. Some of the latter are then calculated by the uniqueness method, others by the Gegenbauer polynomial technique. The result is independent of the ultraviolet and the infrared scale. It is expressed in terms of the squares of spacetime intervals between points of the effective fields in the position space – it includes simple powers of these intervals, as well as logarithms and polylogarithms thereof, with some of the latter appearing within the Davydychev integral  $J(1, 1, 1)$ .

Keywords: Gegenbauer polynomial technique, Davydychev integral  $J(1, 1, 1)$

# 1 Introduction

It has been shown in Refs. [1, 2, 3] that the effective action of dressed mean fields for  $\mathcal{N} = 4$  super-Yang-Mills theory does not depend on any scale, ultraviolet or infrared. These results were derived from the results of Refs. [4, 5, 6, 7, 8]. Scale independence suggests that kernels of these dressed mean fields can be analyzed by the methods of conformal field theory. We started to investigate the simplest scalar vertex  $Lcc$  in the Landau gauge in Ref. [9], where  $c$  is a real ghost field and  $L$  is the auxiliary field which couples at the tree level to the BRST [10, 11] transformation of the  $c$  field. This vertex is simple in the Landau gauge where it is totally finite for  $\mathcal{N} = 4$  super-Yang-Mills theory. By solving Slavnov-Taylor (ST) identity [12, 13, 14, 15, 16, 17], all other vertices in that theory can be derived from this vertex. The ST identity is a consequence of the BRST symmetry of the classical action [10, 11]. Recently, by using unitarity methods it has been demonstrated up to the four-loop level that the only off-shell conformal integrals in the momentum space contribute in the maximally-helicity-violating four-particle amplitudes [18], and iterative structure has been conjectured for all the maximally-helicity-violating amplitudes [19].

To solve the ST identity and to check the conformal invariance explicitly, we should work in the position space. The two-loop contribution contains five diagrams. We calculated the first contribution in the previous paper [9]. Now we calculate the second contribution that corresponds to the diagram (b) of Ref. [9]. The notation used here is the same as in Ref. [9]. The  $Lcc$  vertex is superficially convergent in the Landau gauge. This fact can be checked by index counting and by noting that two derivatives from the ghost propagators can be always integrated out of the diagram due to the transversality of the gauge propagator. It means that the field  $c$  does not have renormalization in the Landau gauge. Formally, this result holds to all orders of perturbation theory due to the so called antighost equation [20]. In the nonsupersymmetric theories this vertex is not finite and a calculation of the anomalous dimension of operator  $cc$  has been performed in [21, 22].

Knowing the structure of the  $Lcc$  vertex one can obtain the structure of other irreducible vertices by solving the ST identity. The algorithm can be applied also to other theories different from the theory in consideration. Among possible applications are  $\mathcal{N} = 8$  supergravity, Chern-Simons theory, string field theory, massless gauge theory near fixed points in the coupling space, and topological field theories in higher dimensions.

The paper has the following structure. In Sec. 2 we write the integral expression that corresponds to the diagram (b), and analyze it by dividing it in three parts. In Sec. 3, the result for the first part is written in terms of single  $D$ -dimensional integrals ( $D = 4 - 2\epsilon$ ). The analogous expressions for the second and the third part are written in Appendix A. The sum of all three parts contains single  $D$ -dimensional integrals  $J(\alpha_1, \alpha_2, \alpha_3)$  with the sum of indices  $\alpha_1 + \alpha_2 + \alpha_3$  in the denominators equal to  $D - 1$ ,  $D$ ,  $D + 1$ ,  $D + 2$ . In Sec. 4 we present the part with the integrals whose sum of indices is equal to  $D - 1$ . In Appendix B we present the part with the integrals whose sum of the indices is  $D$ ,  $D + 1$ , and  $D + 2$ ; these integrals are calculated there explicitly by using the uniqueness method and its variants [23, 24]. In Sec. 4 we reduce the aforementioned  $(D - 1)$ -type partial sum to explicitly known terms and to terms containing the integrals  $J(1, 1, 1)$ ,  $J(\epsilon, 2 - 3\epsilon, 1)$  and derivatives thereof – by applying formulas obtained in Appendix C where the method of integration by parts procedure (IBP) [25, 26] was

used. In Sec. 5 we obtain the entire result for the diagram (b) in  $D$  dimensions, in known terms and terms with the integrals  $J(1, 1, 1)$ ,  $J(\epsilon, 2 - 3\epsilon, 1)$  and derivatives thereof. The terms with  $J(\epsilon, 2 - 3\epsilon, 1)$  are calculated explicitly in Sec. 5, while derivatives of  $J(\epsilon, 2 - 3\epsilon, 1)$  are calculated in Appendix D – in both cases those formulas of Appendix C were employed which were obtained by the Gegenbauer polynomial technique (GPT) introduced originally in Refs. [27, 28, 29, 30] and further developed in Ref. [31]. In Appendix E, all the contributions for the diagram (b) in  $D$  dimensions are collected, and the  $\epsilon \rightarrow 0$  limit is performed. In Sec. 6 we present this result in a more explicit and shorter form. In Conclusions, we comment on the result obtained and on further possible developments of the analysis performed in this paper.

## 2 Diagram (b)

The two-loop correction to the  $Lcc$  vertex can be represented as a sum of five diagrams depicted in Fig. 1. We have introduced for brevity the notation

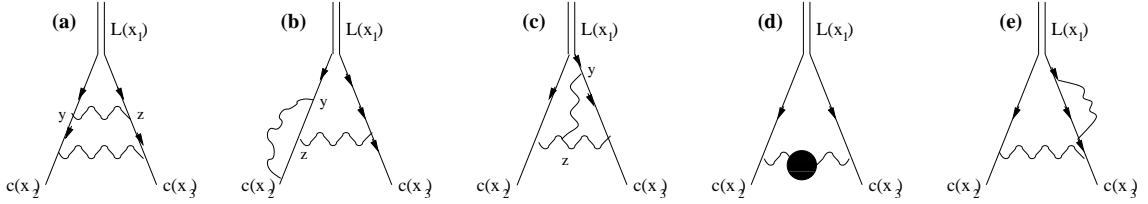


Figure 1: The two-loop diagrams for the  $Lcc$  vertex. The wavy lines represent the gluons, the straight lines the ghosts. Black disk stands for the total one-loop correction to the gluon propagator.

$$[yz] = (y - z)^2, \quad [y1] = (y - x_1)^2, \dots$$

and so on. The structure of Lorentz indices in diagram (b) is more complicated than the structure of the indices in the diagram (a) considered in Ref. [9]. The algebra can be inferred immediately from the diagram (b).

$$\frac{(2z)_\sigma}{[2z]^{2-\epsilon}} \left( \frac{g_{\sigma\lambda}}{[2y]^{1-\epsilon}} + 2(1-\epsilon) \frac{(2y)_\sigma(2y)_\lambda}{[2y]^{2-\epsilon}} \right) \frac{(y1)_\lambda}{[y1]^{2-\epsilon}} \frac{(zy)_\mu}{[zy]^{2-\epsilon}} \left( \frac{g_{\mu\nu}}{[3z]^{1-\epsilon}} + 2(1-\epsilon) \frac{(3z)_\mu(3z)_\nu}{[3z]^{2-\epsilon}} \right) \frac{(31)_\nu}{[31]^{2-\epsilon}}$$

We work in the position space, in  $D = 4 - 2\epsilon$  dimensions, using the technique of dimensional regularization [32], thus maintaining the intermediate results finite. We have to calculate the  $2D$ -dimensional integral<sup>1</sup>

$$\int Dy Dz \frac{(2z)_\sigma}{[2z]^{2-\epsilon}} \left( \frac{g_{\sigma\lambda}}{[2y]^{1-\epsilon}} + 2(1-\epsilon) \frac{(2y)_\sigma(2y)_\lambda}{[2y]^{2-\epsilon}} \right) \frac{(y1)_\lambda}{[y1]^{2-\epsilon}} \times \\ \times \frac{(zy)_\mu}{[zy]^{2-\epsilon}} \left( \frac{g_{\mu\nu}}{[3z]^{1-\epsilon}} + 2(1-\epsilon) \frac{(3z)_\mu(3z)_\nu}{[3z]^{2-\epsilon}} \right) \frac{(31)_\nu}{[31]^{2-\epsilon}}. \quad (1)$$

<sup>1</sup>We use the  $D$ -dimensional measure  $Dx \equiv \pi^{-\frac{D}{2}} d^D x$  [9].

By using the representation  $(y1)_\lambda = (y2)_\lambda + (21)_\lambda$ , after simple algebra we can represent this integral as a sum of three integrals

$$\begin{aligned} & -(3-2\epsilon) \int Dy Dz \frac{(2z)_\sigma (2y)_\sigma}{[y1]^{2-\epsilon} [2z]^{2-\epsilon} [2y]^{1-\epsilon}} \frac{(zy)_\mu}{[zy]^{2-\epsilon}} \left( \frac{g_{\mu\nu}}{[3z]^{1-\epsilon}} + 2(1-\epsilon) \frac{(3z)_\mu (3z)_\nu}{[3z]^{2-\epsilon}} \right) \frac{(31)_\nu}{[31]^{2-\epsilon}} \\ & + (2-2\epsilon) \int Dy Dz \frac{[(2z)_\sigma (2y)_\sigma][(2y)_\lambda (21)_\lambda]}{[y1]^{2-\epsilon} [2z]^{2-\epsilon} [2y]^{1-\epsilon}} \frac{(zy)_\mu}{[zy]^{2-\epsilon}} \left( \frac{g_{\mu\nu}}{[3z]^{1-\epsilon}} + 2(1-\epsilon) \frac{(3z)_\mu (3z)_\nu}{[3z]^{2-\epsilon}} \right) \frac{(31)_\nu}{[31]^{2-\epsilon}} \\ & + \int Dy Dz \frac{(2z)_\sigma (21)_\sigma}{[y1]^{2-\epsilon} [2z]^{2-\epsilon} [2y]^{1-\epsilon}} \frac{(zy)_\mu}{[zy]^{2-\epsilon}} \left( \frac{g_{\mu\nu}}{[3z]^{1-\epsilon}} + 2(1-\epsilon) \frac{(3z)_\mu (3z)_\nu}{[3z]^{2-\epsilon}} \right) \frac{(31)_\nu}{[31]^{2-\epsilon}}. \end{aligned}$$

As in the previous paper [9], we modify the  $\epsilon$ 's appearing in the propagators ( $\epsilon \mapsto \kappa_j \epsilon$ ) in order to make the uniqueness method applicable and thus to make the calculations easier. Such changes do not affect our result in the  $\epsilon \rightarrow 0$  limit since the integral is finite. We do the following changes: in the powers of the denominators of the four ghost propagators in Eq. (1), we change  $\epsilon \mapsto 2\epsilon$  in the first one, and  $\epsilon \mapsto 0$  in the other three; in the powers of the denominators in the two gauge propagators, we change  $\epsilon \mapsto 2\epsilon$  in the first and  $\epsilon \mapsto 0$  in the second propagator; in the nominator of the second gauge propagator, we change  $\epsilon \mapsto \bar{\epsilon}$ , where  $\bar{\epsilon}$  is as in the previous paper; in the nominator of the first gauge propagator, we change  $\epsilon \mapsto \bar{\epsilon}_2$  where  $\bar{\epsilon}_2$  is different because it is related to the requirement of keeping the transversality in the modified propagators [9]

$$\bar{\epsilon} = -\frac{2\epsilon}{(1-2\epsilon)}, \quad \bar{\epsilon}_2 = \frac{4\epsilon}{(1+2\epsilon)}. \quad (2)$$

The above integral for the diagram (b) now acquires the following form:

$$\begin{aligned} I_b &= \int Dy Dz \frac{(2z)_\sigma}{[2z]^{2-2\epsilon}} \left( \frac{g_{\sigma\lambda}}{[2y]^{1-2\epsilon}} + 2(1-\bar{\epsilon}_2) \frac{(2y)_\sigma (2y)_\lambda}{[2y]^{2-2\epsilon}} \right) \frac{(y1)_\lambda}{[y1]^2} \\ &\quad \times \frac{(zy)_\mu}{[zy]^2} \left( \frac{g_{\mu\nu}}{[3z]} + 2(1-\bar{\epsilon}) \frac{(3z)_\mu (3z)_\nu}{[3z]^2} \right) \frac{(31)_\nu}{[31]^2} \\ &= -(3-2\bar{\epsilon}_2) I_{b,1} + (2-2\bar{\epsilon}_2) I_{b,2} + I_{b,3}, \end{aligned} \quad (3)$$

where the three integrals  $I_{b,j}$  are

$$I_{b,1} = \int Dy Dz \frac{(2z)_\sigma (2y)_\sigma}{[y1]^2 [2z]^{2-2\epsilon} [2y]^{1-2\epsilon}} \frac{(zy)_\mu}{[zy]^2} \left( \frac{g_{\mu\nu}}{[3z]} + 2(1-\bar{\epsilon}) \frac{(3z)_\mu (3z)_\nu}{[3z]^2} \right) \frac{(31)_\nu}{[31]^2}, \quad (4)$$

$$I_{b,2} = \int Dy Dz \frac{[(2z)_\sigma (2y)_\sigma][(2y)_\lambda (21)_\lambda]}{[y1]^2 [2z]^{2-2\epsilon} [2y]^{2-2\epsilon}} \frac{(zy)_\mu}{[zy]^2} \left( \frac{g_{\mu\nu}}{[3z]} + 2(1-\bar{\epsilon}) \frac{(3z)_\mu (3z)_\nu}{[3z]^2} \right) \frac{(31)_\nu}{[31]^2}, \quad (5)$$

$$I_{b,3} = \int Dy Dz \frac{(2z)_\sigma (21)_\sigma}{[y1]^2 [2z]^{2-2\epsilon} [2y]^{1-2\epsilon}} \frac{(zy)_\mu}{[zy]^2} \left( \frac{g_{\mu\nu}}{[3z]} + 2(1-\bar{\epsilon}) \frac{(3z)_\mu (3z)_\nu}{[3z]^2} \right) \frac{(31)_\nu}{[31]^2}. \quad (6)$$

In these  $2D$ -dimensional integrals, we first perform the Lorentz algebra only for certain subproducts in the numerators, then perform one of the two  $D$ -dimensional integrations by using the uniqueness method or its variants,<sup>2</sup> and then finish the Lorentz algebra. We found this method simpler than making the entire Lorentz algebra initially and then integrating the contributions. After integrating, the number of terms in the algebra is significantly reduced.

---

<sup>2</sup>By the ‘‘variants of the uniqueness method’’ we mean the cases when the sum of the powers in the denominator of the integrand is larger than  $D$  but can be made equal  $D$  once the integrand is represented as a derivative, or derivatives, with respect to a coordinate that is not integrated over.

### 3 Reduction to single scalar integrals

Following the afore-described procedure, we can scalarize the expression for each of the three  $2D$ -dimensional integrals in Eq. (3), reducing them to linear combinations of integrals  $J(\alpha_1, \alpha_2, \alpha_3)$  where

$$J(\alpha_1, \alpha_2, \alpha_3) = \int Dx \frac{1}{[x1]^{\alpha_1} [x2]^{\alpha_2} [x3]^{\alpha_3}} .$$

For the first integral we obtain

$$\begin{aligned} I_{b,1} = & \frac{A(1, 1-2\epsilon, 1)}{4\epsilon(1-2\epsilon)^2} \left\{ \left( -\frac{6(1-\epsilon^2)(1-2\epsilon)}{[31]^2[12]^{-\epsilon}} - \frac{2(1-\epsilon^2)(1-2\epsilon)}{[31][12]^{1-\epsilon}} \right) J(1+\epsilon, 2-3\epsilon, 1) \right. \\ & + \frac{2(1-\epsilon^2)(1-2\epsilon)}{[31]^2[12]^{1-\epsilon}} J(\epsilon, 2-3\epsilon, 1) + \frac{2(1-\epsilon^2)(1-2\epsilon)}{[31][12]^{1-\epsilon}} J(2+\epsilon, 1-3\epsilon, 1) \\ & + \frac{2(1-\epsilon^2)(1-2\epsilon)}{[31]^2[12]^{1-\epsilon}} J(2+\epsilon, 1-3\epsilon, 0) - \frac{(1-2\epsilon^2)(1-2\epsilon)}{[31]^2[12]^{1-\epsilon}} J(1+\epsilon, 1-3\epsilon, 1) \\ & + \frac{6(1-\epsilon^2)(1-2\epsilon)}{[31][12]^{-\epsilon}} J(2+\epsilon, 2-3\epsilon, 1) - \frac{2(1-\epsilon^2)(1-2\epsilon)}{[31]^2[12]^{1-\epsilon}} J(1+\epsilon, 2-3\epsilon, 0) \\ & + \frac{\epsilon-\epsilon^2}{[31][23]^{1-\epsilon}} J(2, 1-3\epsilon, 1+\epsilon) + \frac{\epsilon-\epsilon^2}{[31]^2[23]^{1-\epsilon}} J(1, 2-3\epsilon, \epsilon) \\ & - \frac{4-8\epsilon+3\epsilon^2}{[31][23]^{-\epsilon}} J(2, 2-3\epsilon, 1+\epsilon) + \frac{4-8\epsilon+3\epsilon^2}{[31]^2[23]^{-\epsilon}} J(1, 2-3\epsilon, 1+\epsilon) \\ & + \left( \frac{\epsilon(1-2\epsilon)}{[31][23]^{1-\epsilon}} - \frac{4-10\epsilon+6\epsilon^2}{[31]^2[23]^{-\epsilon}} - \frac{[12]\epsilon(2-3\epsilon)}{[31]^2[23]^{1-\epsilon}} \right) J(2, 2-3\epsilon, \epsilon) \\ & + \left[ -\frac{(1-2\epsilon)(1-\epsilon)}{[31]^2[23]^{1-\epsilon}} - \frac{(1-\epsilon)^2}{[31][23]^{2-\epsilon}} + \frac{[12](1-\epsilon)^2}{[31]^2[23]^{2-\epsilon}} \right] J(2, 1-3\epsilon, \epsilon) \\ & + \left[ -\frac{2\epsilon(1-\epsilon)}{[31]^2[23]^{1-\epsilon}} - \frac{\epsilon(1-\epsilon)}{[31][23]^{2-\epsilon}} + \frac{[12]\epsilon(1-\epsilon)}{[31]^2[23]^{2-\epsilon}} \right] J(2, 2-3\epsilon, \epsilon-1) \\ & + \frac{1-2\epsilon}{[12]^{1-\epsilon}[31]} J(1+\epsilon, 1-3\epsilon, 2) - \frac{1-2\epsilon}{[12]^{1-\epsilon}[31]^2} J(\epsilon, 1-3\epsilon, 2) \\ & + \left( \frac{1-\epsilon}{[31][23]^{2-\epsilon}} + \frac{1-\epsilon}{[31]^2[23]^{1-\epsilon}} - \frac{[12](1-\epsilon)}{[31]^2[23]^{2-\epsilon}} \right) J(2, 1-2\epsilon, 0) \\ & \left. - \frac{\epsilon-\epsilon^2}{[31]^2[23]^{1-\epsilon}} J(1, 1-3\epsilon, 1+\epsilon) \right\} . \quad (7) \end{aligned}$$

Here we use the notation

$$A(\alpha_1, \alpha_2, \alpha_3) = a(\alpha_1)a(\alpha_2)a(\alpha_3); \quad a(\alpha) = \frac{\Gamma(D/2 - \alpha)}{\Gamma(\alpha)} .$$

The second and the third integrals (5) and (6) are given in Appendix A, Eqs. (A.1) and (A.2), respectively, and have a similar structure as the first integral (7). As a consequence, the total

Integral  $I_b$  can be organized into the sum

$$I_b = \sum_{k=D-1}^{D+2} I_b^{(k)}, \quad (8)$$

where  $I_b^{(k)}$  represents the terms with  $J(\alpha_1, \alpha_2, \alpha_3)$  with  $\alpha_1 + \alpha_2 + \alpha_3 = k$ .

#### 4 Integrals with the sum of indices $D - 1$

The result for  $I_b^{(D)}$  and for  $I_b^{(D+1)} + I_b^{(D+2)}$  is written in Appendix B in terms of  $D$ -dimensional  $J$ -integrals and then reduced to explicit expressions by using the uniqueness relation and its variants for the integrals. Below we consider the sum  $I_b^{(D-1)}$ .

$$\begin{aligned} I_b^{(D-1)} = & \frac{A(1, 1 - 2\epsilon, 1)}{4\epsilon(1 + 2\epsilon)(1 - 2\epsilon)^2} \left\{ \frac{-1 + 5\epsilon - 6\epsilon^2}{[31]^2[12]^{1-\epsilon}} J(\epsilon, 2 - 3\epsilon, 1) \right. \\ & + \left[ \frac{1 - 7\epsilon + 10\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{1 - 4\epsilon + 5\epsilon^2}{[31][23]^{2-\epsilon}} - \frac{(1 - 4\epsilon + 5\epsilon^2)[21]}{[31]^2[23]^{2-\epsilon}} \right] J(1, 2 - 3\epsilon, \epsilon) \\ & + \left[ \frac{2 - 6\epsilon + 4\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{2 - 4\epsilon + 2\epsilon^2}{[31][23]^{2-\epsilon}} - \frac{[12](2 - 4\epsilon + 2\epsilon^2)}{[31]^2[23]^{2-\epsilon}} \right] J(2, 1 - 3\epsilon, \epsilon) \\ & + \left[ \frac{4\epsilon - 4\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{2\epsilon - 2\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{[12](-2\epsilon + 2\epsilon^2)}{[31]^2[23]^{2-\epsilon}} \right] J(2, 2 - 3\epsilon, \epsilon - 1) \\ & + \frac{-\epsilon + 2\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(\epsilon, 1 - 3\epsilon, 2) \\ & + \frac{1 - \epsilon - 2\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(\epsilon - 1, 2 - 3\epsilon, 2) \\ & + \frac{-2\epsilon + 4\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(\epsilon - 1, 3 - 3\epsilon, 1) \\ & \left. + \left( \frac{2\epsilon - 6\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{\epsilon - 3\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{(-\epsilon + 3\epsilon^2)[12]}{[31]^2[23]^{2-\epsilon}} \right) J(1, 3 - 3\epsilon, \epsilon - 1) \right\}. \quad (9) \end{aligned}$$

The  $D$ -dimensional integrals appearing in  $I_b^{(D-1)}$  cannot be found by the uniqueness method or its variants. We will reduce them in this Section to explicit expressions and to terms proportional to the integrals  $J(1, 1, 1)$ ,  $J(\epsilon, 2 - 3\epsilon, 1)$ ,  $J(1, 2 - 3\epsilon, \epsilon)$  and specific derivatives thereof – by applying the integration by parts (IBP) procedure [25, 26]. Using formula (C.1) from Appendix C (IBP), we represent integral (9) as

$$\begin{aligned} I_b^{(D-1)} = & \frac{A(1, 1 - 2\epsilon, 1)}{4\epsilon(1 + 2\epsilon)(1 - 2\epsilon)^2} \left\{ \frac{-1 + 4\epsilon - (9/2)\epsilon^2}{[31]^2[12]^{1-\epsilon}} J(\epsilon, 2 - 3\epsilon, 1) \right. \\ & + \left[ \frac{1 - 6\epsilon + (15/2)\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{1 - 7/2\epsilon + 15/4\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{(-1 + 7/2\epsilon - 15/4\epsilon^2)[21]}{[31]^2[23]^{2-\epsilon}} \right] J(1, 2 - 3\epsilon, \epsilon) \end{aligned}$$

$$\begin{aligned}
& + \left[ \frac{2-6\epsilon+4\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{2-4\epsilon+2\epsilon^2}{[31][23]^{2-\epsilon}} - \frac{[12](2-4\epsilon+2\epsilon^2)}{[31]^2[23]^{2-\epsilon}} \right] J(2, 1-3\epsilon, \epsilon) \\
& \quad + \frac{-\epsilon+2\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(\epsilon, 1-3\epsilon, 2) \\
& + \left[ \frac{3\epsilon-(5/2)\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{(3/2)\epsilon-(5/4)\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{[12](-(3/2)\epsilon+(5/4)\epsilon^2)}{[31]^2[23]^{2-\epsilon}} \right] J(2, 2-3\epsilon, \epsilon-1) \\
& \quad + \frac{1-(5/2)\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(\epsilon-1, 2-3\epsilon, 2) \Big\} \\
& + \frac{A(1, 1-2\epsilon, 1)A(1, 1-3\epsilon, 1+\epsilon)}{8\epsilon^2(1+2\epsilon)(1-2\epsilon)^2} \left( \frac{-\epsilon-(1/2)\epsilon^2}{[12]^{2-2\epsilon}[23]^{1-2\epsilon}[31]^{1+2\epsilon}} + \frac{-(\epsilon/2)-(1/4)\epsilon^2}{[12]^{2-2\epsilon}[23]^{2-2\epsilon}[31]^{2\epsilon}} \right. \\
& \quad \left. + \frac{(3/2)\epsilon+(7/4)\epsilon^2}{[12]^{1-2\epsilon}[23]^{2-2\epsilon}[31]^{1+2\epsilon}} \right).
\end{aligned}$$

After applying formulas (C.3) and (C.4) (IBP), this formula takes the following form:

$$\begin{aligned}
I_b^{(D-1)} &= \frac{A(1, 1-2\epsilon, 1)}{4\epsilon(1+2\epsilon)(1-2\epsilon)^2} \left\{ \frac{-1+5\epsilon-(19/2)\epsilon^2}{[31]^2[12]^{1-\epsilon}} J(\epsilon, 2-3\epsilon, 1) \right. \\
& + \left[ \frac{-1+6\epsilon-(29/2)\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{-1+13/2\epsilon-41/4\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{(1-13/2\epsilon+41/4\epsilon^2)[21]}{[31]^2[23]^{2-\epsilon}} \right] J(1, 2-3\epsilon, \epsilon) \\
& + \left[ \frac{3\epsilon-(5/2)\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{(3/2)\epsilon-(5/4)\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{[12](-(3/2)\epsilon+(5/4)\epsilon^2)}{[31]^2[23]^{2-\epsilon}} \right] J(2, 2-3\epsilon, \epsilon-1) \\
& \quad \left. + \frac{1-(5/2)\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(\epsilon-1, 2-3\epsilon, 2) \right\} \\
& - \frac{1}{2} \left[ \frac{1}{[31]^2[23]} + \frac{1}{[31][23]^2} - \frac{[12]}{[31]^2[23]^2} \right] J(1, 1, 1) \\
& + \frac{A(1, 1-2\epsilon, 1)A(1, 1-3\epsilon, 1+\epsilon)}{8\epsilon^2(1+2\epsilon)(1-2\epsilon)^2} \left[ \frac{-2+7\epsilon-6\epsilon^2}{[12]^{1-2\epsilon}[23]^{1-2\epsilon}[31]^{2+2\epsilon}} + \frac{-2+(11/2)\epsilon-(1/4)\epsilon^2}{[12]^{1-2\epsilon}[23]^{2-2\epsilon}[31]^{1+2\epsilon}} \right. \\
& \quad \left. + \frac{(2-4\epsilon+2\epsilon^2)}{[12]^{-2\epsilon}[23]^{2-2\epsilon}[31]^{2+2\epsilon}} + \frac{-\epsilon-(1/2)\epsilon^2}{[12]^{2-2\epsilon}[23]^{1-2\epsilon}[31]^{1+2\epsilon}} + \frac{-(\epsilon/2)-(1/4)\epsilon^2}{[12]^{2-2\epsilon}[23]^{2-2\epsilon}[31]^{2\epsilon}} \right].
\end{aligned}$$

Finally, applying relations (C.5) derived in Appendix C, we obtain the representation

$$\begin{aligned}
I_b^{(D-1)} &= \frac{A(1, 1-2\epsilon, 1)}{4\epsilon(1+2\epsilon)(1-2\epsilon)^2} \left\{ \frac{5\epsilon-12\epsilon^2}{[31]^2[12]^{1-\epsilon}} J(\epsilon, 2-3\epsilon, 1) \right. \\
& + \left[ \frac{-1+9\epsilon-17\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{-1+8\epsilon-(23/2)\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{(1-8\epsilon+(23/2)\epsilon^2)[21]}{[31]^2[23]^{2-\epsilon}} \right] J(1, 2-3\epsilon, \epsilon) \\
& + \left[ \frac{3\epsilon-(5/2)\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{-(3/2)\epsilon+(5/4)\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{[12](-(3/2)\epsilon+(5/4)\epsilon^2)}{[31]^2[23]^{2-\epsilon}} \right] (13)_\mu \partial_\mu^{(1)} J(1, 2-3\epsilon, \epsilon)
\end{aligned}$$

$$\begin{aligned}
& + \frac{1 - (5/2)\epsilon^2}{[12]^{1-\epsilon}[31]^2} (31)_\mu \partial_\mu^{(3)} J(\epsilon, 2 - 3\epsilon, 1) \Big\} \\
& - \frac{1}{2} \left[ \frac{1}{[31]^2[23]} + \frac{1}{[31][23]^2} - \frac{[12]}{[31]^2[23]^2} \right] J(1, 1, 1) \\
& + \frac{A(1, 1 - 2\epsilon, 1)A(1, 1 - 3\epsilon, 1 + \epsilon)}{8\epsilon^2(1 + 2\epsilon)(1 - 2\epsilon)^2} \left[ \frac{-2 + 7\epsilon - 6\epsilon^2}{[12]^{1-2\epsilon}[23]^{1-2\epsilon}[31]^{2+2\epsilon}} + \frac{-3 + 6\epsilon - 1/2\epsilon^2}{[12]^{1-2\epsilon}[23]^{2-2\epsilon}[31]^{1+2\epsilon}} \right. \\
& \left. + \frac{2 - 4\epsilon + 2\epsilon^2}{[12]^{-2\epsilon}[23]^{2-2\epsilon}[31]^{2+2\epsilon}} + \frac{-4\epsilon - \epsilon^2}{[12]^{2-2\epsilon}[23]^{1-2\epsilon}[31]^{1+2\epsilon}} + \frac{-2\epsilon - \epsilon^2/2}{[12]^{2-2\epsilon}[23]^{2-2\epsilon}[31]^{2\epsilon}} \right] . \quad (10)
\end{aligned}$$

## 5 The sum

Summing up Eqs. (10), (B.1) and (B.2), we obtain

$$\begin{aligned}
I_b = & \frac{A(1, 1 - 2\epsilon, 1)}{4\epsilon(1 + 2\epsilon)(1 - 2\epsilon)^2} \left\{ \frac{5\epsilon - 12\epsilon^2}{[31]^2[12]^{1-\epsilon}} J(\epsilon, 2 - 3\epsilon, 1) \right. \\
& + \left[ \frac{-1 + 9\epsilon - 17\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{-1 + 8\epsilon - (23/2)\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{(1 - 8\epsilon + (23/2)\epsilon^2)[21]}{[31]^2[23]^{2-\epsilon}} \right] J(1, 2 - 3\epsilon, \epsilon) \\
& + \left[ \frac{3\epsilon - (5/2)\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{-(3/2)\epsilon + (5/4)\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{[12](-(3/2)\epsilon + (5/4)\epsilon^2)}{[31]^2[23]^{2-\epsilon}} \right] (13)_\mu \partial_\mu^{(1)} J(1, 2 - 3\epsilon, \epsilon) \\
& \left. + \frac{1 - (5/2)\epsilon^2}{[12]^{1-\epsilon}[31]^2} (31)_\mu \partial_\mu^{(3)} J(\epsilon, 2 - 3\epsilon, 1) \right\} \\
& - \frac{1}{2} \left[ \frac{1}{[31]^2[23]} + \frac{1}{[31][23]^2} - \frac{[12]}{[31]^2[23]^2} \right] J(1, 1, 1) \\
& + \frac{1}{8} \left\{ \frac{-19/4}{[12][23][31]^2} + \frac{5}{[12][23]^2[31]} + \frac{1/4}{[12]^2[23][31]} + \frac{-11/4}{[12]^2[23]^2} + \frac{-9/4}{[23]^2[31]^2} + \frac{5/2}{[12]^2[31]^2} \right\} \\
& + \frac{A(1, 1 - 2\epsilon, 1)A(1, 1 - 3\epsilon, 1 + \epsilon)}{8\epsilon(1 + 2\epsilon)(1 - 2\epsilon)^2} \left\{ \frac{-7/2}{[12]^{1-2\epsilon}[23]^{1-2\epsilon}[31]^{2+2\epsilon}} + \frac{-2}{[12]^{1-2\epsilon}[23]^{2-2\epsilon}[31]^{1+2\epsilon}} \right. \\
& + \frac{-1/2}{[12]^{-2\epsilon}[23]^{2-2\epsilon}[31]^{2+2\epsilon}} + \frac{3/2}{[12]^{2-2\epsilon}[23]^{1-2\epsilon}[31]^{1+2\epsilon}} + \frac{3/2}{[12]^{2-2\epsilon}[23]^{2-2\epsilon}[31]^{2\epsilon}} \\
& \left. + \frac{3}{[12]^{2-2\epsilon}[23]^{-2\epsilon}[31]^{2+2\epsilon}} + \frac{-6}{[31]^2[12]^{2-2\epsilon}} + \frac{1}{[31]^2[23]^{2-2\epsilon}} \right\} \\
& + \frac{A^2(1, 1 - 2\epsilon, 1)}{4\epsilon^2(1 + 2\epsilon)(1 - 2\epsilon)^2} \left\{ \frac{1 - \epsilon}{[12]^{1-\epsilon}[23]^{2-\epsilon}[31]} + \frac{1 - \epsilon}{[12]^{1-\epsilon}[23]^{1-\epsilon}[31]^2} - \frac{1 - \epsilon}{[12]^{-\epsilon}[23]^{2-\epsilon}[31]^2} \right\} . \quad (11)
\end{aligned}$$

After applying formulas (C.6) which were obtained by an application of the Gegenbauer polynomial technique (GPT, cf. Ref. [31]), we obtain for the terms proportional to  $J(\epsilon, 2 - 3\epsilon, 1)$  and  $J(1, 2 - 3\epsilon, \epsilon)$  in  $I_b$  the following explicit result:

$$I_b^{(J)} \equiv \frac{A(1, 1 - 2\epsilon, 1)}{4\epsilon(1 + 2\epsilon)(1 - 2\epsilon)^2} \left\{ \frac{5\epsilon - 12\epsilon^2}{[31]^2[12]^{1-\epsilon}} J(\epsilon, 2 - 3\epsilon, 1) \right.$$



$$\begin{aligned}
& + \left[ \frac{-1 + 9\epsilon - 17\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{-1 + 8\epsilon - (23/2)\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{(1 - 8\epsilon + (23/2)\epsilon^2)[21]}{[31]^2[23]^{2-\epsilon}} \right] J(1, 2 - 3\epsilon, \epsilon) \Big\} \\
& = \frac{A(1, 1 - 2\epsilon, 1)A(1 - 3\epsilon, 1, 1 + \epsilon)}{8\epsilon(1 + 2\epsilon)(1 - 2\epsilon)^2} \left[ \frac{11}{[12]^{1-\epsilon}[31]^2[23]^{1-\epsilon}} \right. \\
& \quad \left. + \frac{5}{[12]^{1-\epsilon}[31][23]^{2-\epsilon}} + \frac{-5}{[12]^{-\epsilon}[31]^2[23]^{2-\epsilon}} \right] \\
& + \frac{A(1, 1 - 2\epsilon, 1)A(1 - 3\epsilon, 1, 1 + \epsilon)}{8\epsilon^2(1 + 2\epsilon)(1 - 2\epsilon)^2} \left[ \frac{-1}{[12]^{1-\epsilon}[31]^2[23]^{1-\epsilon}} \right. \\
& \quad \left. + \frac{-1}{[12]^{1-\epsilon}[31][23]^{2-\epsilon}} + \frac{1}{[12]^{-\epsilon}[31]^2[23]^{2-\epsilon}} \right] \\
& + \frac{A(1, 1 - 2\epsilon, 1)A(1 - 3\epsilon, 1, 1 + \epsilon)}{8\epsilon(1 + 2\epsilon)(1 - 2\epsilon)^2} \left[ \frac{1}{[12]^{1-\epsilon}[31]^2[23]^{1-\epsilon}} \right. \\
& \quad \left. + \frac{1}{[12]^{1-\epsilon}[31][23]^{2-\epsilon}} - \frac{1}{[12]^{-\epsilon}[31]^2[23]^{2-\epsilon}} \right] \ln \frac{[23]}{[12]} \\
& \quad + \frac{1}{8} \left[ \frac{4}{[12][31]^2[23]} + \frac{7/2}{[12][31][23]^2} + \frac{-(7/2)}{[31]^2[23]^2} \right] \\
& \quad - \frac{1}{8} \left[ \frac{1}{[12][31]^2[23]} + \frac{5}{[12][31][23]^2} + \frac{-5}{[31]^2[23]^2} \right] \ln \frac{[23]}{[12]} \\
& + \frac{1}{16} \left[ \frac{1}{[12][31]^2[23]} + \frac{1}{[12][31][23]^2} + \frac{-1}{[31]^2[23]^2} \right] \ln \frac{[12][23]}{[13]^2} \ln \frac{[23]}{[12]} \\
& \quad + \frac{1}{8} \left[ \frac{2}{[31]^2[23]} - \frac{[12]}{[31]^2[23]^2} + \frac{1}{[12][23]^2} - \frac{1}{[31]^2[12]} \right] J(1, 1, 1) . \quad (12)
\end{aligned}$$

The explicit results for the terms  $I_b^{(\partial J, 1)}, I_b^{(\partial J, 2)}$  proportional to derivatives of  $J(1, 2 - 3\epsilon, \epsilon)$  and  $J(\epsilon, 2 - 3\epsilon, 1)$  in  $I_b$  are written in Appendix D in Eqs. (D.1) and (D.2), respectively.

## 6 The final result

Substitution of expressions (12), (D.1) and (D.2) into the sum (11), and its expansion in terms of  $\epsilon$ , is performed in Appendix E, Eq. (E.2). Performing explicitly the derivatives appearing in Eq. (E.2), we obtain the final result for the diagram (b):

$$\begin{aligned}
I_b(\epsilon \rightarrow 0) & = \frac{1}{8} \left[ \frac{-3 \ln[23] - 4 \ln[12] + 7 \ln[13]}{[12][23][31]^2} + \frac{-8 \ln[23] + 4 \ln[12] + 4 \ln[13]}{[12][23]^2[31]} \right. \\
& \quad + \frac{3 \ln[23] - 3 \ln[13]}{[12]^2[23][31]} + \frac{5 \ln[23] - 6 \ln[12] + \ln[13]}{[23]^2[31]^2} + \frac{6 \ln[23] - 6 \ln[13]}{[12]^2[31]^2} \\
& \quad \left. + \frac{3 \ln[23] - 3 \ln[13]}{[12]^2[23]^2} \right] \\
& \quad + \frac{1}{4} \left[ \frac{2[12]}{[31]^2[23]^2} + \frac{1}{[12][23]^2} + \frac{-1}{[31]^2[12]} + \frac{-3}{[31][23]^2} \right] J(1, 1, 1)
\end{aligned}$$

$$+\frac{1}{8}\left\{\frac{-2}{[12][23][31]^2}+\frac{8}{[12][23]^2[31]}+\frac{-3}{[12]^2[23]^2}+\frac{-3}{[23]^2[31]^2}+\frac{3}{[12]^2[31]^2}\right\}. \quad (13)$$

The result for the Davydychev integral  $J(1,1,1)$  is in Ref. [33]. A new integral representation for it has been found in Ref. [9].

## 7 Conclusions

In this paper we presented the calculation of the second diagram [diagram (b)] of the five planar two-loop diagrams to the  $Lcc$  vertex depicted in Fig. 1 for a general Yang-Mills theory. The Lorentz structure of the corresponding integrand is complicated in comparison with the integrand of the previously calculated diagram (a) [9]. In order to apply the same methods as in the diagram (a), we first performed in the integrands the Lorentz algebra for certain subproducts in the numerators, then performed one of the two  $D$ -dimensional integrations, then finished the Lorentz algebra, and finally performed the second  $D$ -dimensional integration. It is a long procedure that requires certain computer resources. In principle, it is possible to use another trick to reduce the calculation [53], a trick which we will apply in the future calculation of the diagram (c). Neither the result for the diagram (b) obtained here and nor the result for the diagram (a) obtained in our previous work [9] depend on any scale, infrared or ultraviolet, in complete correspondence with the index counting arguments. By using the ST identity, other (leading- $N_c$ ) two-loop vertices can be derived from the (leading- $N_c$ ) two-loop  $Lcc$  vertex once the latter is known. For example, it will be possible to derive the four-point off-shell correlator, and consequently reproduce the known result for the four-point gluon amplitude [18, 19] and the anomalous dimensions of twist-two operators [34, 35, 36, 37, 38].

It is possible to look at these results from different points of view. On the one hand, the diagrams (d) and (e) are proportional to the one-loop result for the  $Lcc$  correlator written in Ref. [9]. This is due to dimensional considerations. Indeed, since we do not have any dimensionful parameter like meson mass etc., by dimensional considerations the one-loop insertions into the tree-level propagators in the diagrams (d) and (e) are proportional to the tree-level propagators themselves with singular coefficients. Moreover, the transversality of the gauge propagator must be conserved by the radiative correction, since it is a well-known fact that the gauge fixing term does not obtain any quantum correction [15]. This means that the sum of diagrams (d) and (e) is proportional to the one-loop contribution, with a coefficient that is singular in a general nonsupersymmetric massless gauge theory, but in this particular theory it is a finite number since the poles in  $\epsilon$  cancel in the sum of the diagrams (d) and (e) due to the  $\mathcal{N} = 4$  supersymmetry. In the nonsupersymmetric case, that singular number can be absorbed into the gauge coupling to organize the bare coupling. It means that in  $D$  dimensions the massless nonsupersymmetric gauge theory is a conformal gauge theory in terms of the bare coupling and dressed mean fields. In other words, we think that, in terms of the bare couplings and dressed mean fields, the arguments of Ref. [1] can be applied without modifications to the massless QCD. The role of the renormalization group (RG) scale could be the coordinate of the moduli space of the theory.

On the other hand, QCD is asymptotically free theory and for short distances we can consider it as a theory with zero beta-function and the conformal structure could be restored at short distances by the method of conformal theory in terms of dressed mean fields. Further, it would be

interesting to investigate the relation of off-shell correlators of dressed mean fields and correlators of instantons in multi-dimensional theories. To introduce masses in the theory, we need to use softly broken supersymmetry, in which the couplings are spacetime-independent background superfields. The relation between the RG functions of softly broken and rigid theories was found out in Refs. [39, 40, 41]. The relation between the correlators of softly broken and rigid theories can be found by a trick of general change of variables in superspace [42].

To restore the conformal structure of all the effective action, we need to solve the ST identity. Algorithm for solving the ST identity could be applied to various theories, such as the  $\mathcal{N} = 8$  supergravity [43, 44, 45, 46], Chern-Simons theory near the RG fixed points [47], massless gauge theory near fixed points in the coupling space, topological field theories in higher dimensions, finite  $\mathcal{N} = 1$  supersymmetric theories [48, 49, 50, 51, 52]. We further note that, in addition to the five planar two-loop diagrams of Fig. 1, there is one nonplanar diagram (the nonplanar variant of the diagram (a)) which is suppressed in the planar limit of the large 't Hooft coupling by the factor  $1/N_c$  and so it cannot be in competition with the planar diagram for the disappearance of the contribution proportional to  $J(1, 1, 1)$  in the sum of the diagrams (a, b, c).

## Acknowledgments

The work of I.K. was supported by Ministry of Education (Chile) under grant Mecesup FSM9901 and by DGIP UTFSM, by Fondecyt (Chile) grant #1040368, and by Departamento de Ciencias Básicas de la Universidad del Bío-Bío, Chillán (Chile). The work of G.C. was supported by Fondecyt (Chile) grant #1050512. We are grateful to Anatoly Kotikov for checking a part of formulas in the manuscript and to Ivan Schmidt for careful reading of the manuscript.

## Appendix A

In this Appendix we present the result for the second and the third integrals (5) and (6) in terms of  $D$ -dimensional integrals  $J$ . The result for the second integral is

$$\begin{aligned}
I_{b,2} = & \frac{A(1, 1 - 2\epsilon, 1)}{8\epsilon(1 - 2\epsilon)^2} \left\{ \frac{(8 - 20\epsilon)(1 - \epsilon^2)}{[31][12]^{-\epsilon}} J(2 + \epsilon, 2 - 3\epsilon, 1) \right. \\
& + \frac{(1 - \epsilon^2)(4 - 6\epsilon)}{[31][12]^{-1-\epsilon}} J(2 + \epsilon, 3 - 3\epsilon, 1) \\
& + \left[ -\frac{2\epsilon^2(1 - \epsilon)}{[31][12]^{1-\epsilon}} + \frac{2(2 - \epsilon - 4\epsilon^2)(1 - \epsilon)}{[31]^2[12]^{-\epsilon}} \right] J(\epsilon, 3 - 3\epsilon, 1) \\
& + \frac{2(1 - \epsilon^2)(2 - 3\epsilon)}{[31][12]^{1-\epsilon}} J(2 + \epsilon, 1 - 3\epsilon, 1) \\
& - \frac{2(2 - \epsilon - 4\epsilon^2)(1 - \epsilon)}{[31]^2[12]^{1-\epsilon}} J(1 + \epsilon, 2 - 3\epsilon, 0) + \frac{2(1 - \epsilon^2)(2 - 3\epsilon)}{[31]^2[12]^{1-\epsilon}} J(2 + \epsilon, 1 - 3\epsilon, 0) \\
& \left. - \frac{2\epsilon^2(1 - \epsilon)}{[31]^2[12]^{1-\epsilon}} J(\epsilon, 3 - 3\epsilon, 0) \right\}
\end{aligned}$$

$$\begin{aligned}
& + \left[ -\frac{7-17\epsilon-8\epsilon^2+20\epsilon^3}{[31]^2[12]^{-\epsilon}} - \frac{2(2-\epsilon-4\epsilon^2)(1-\epsilon)}{[31][12]^{1-\epsilon}} \right] J(1+\epsilon, 2-3\epsilon, 1) \\
& + \left[ -\frac{2(1-\epsilon^2)(2-3\epsilon)}{[31]^2[12]^{-1-\epsilon}} - \frac{2(2-\epsilon-4\epsilon^2)(1-\epsilon)}{[31][12]^{-\epsilon}} \right] J(1+\epsilon, 3-3\epsilon, 1) \\
& \quad + \frac{4-5\epsilon-6\epsilon^2+8\epsilon^3}{[31]^2[12]^{1-\epsilon}} J(\epsilon, 2-3\epsilon, 1) \\
& + \frac{2\epsilon^2(1-\epsilon)}{[31]^2[12]^{1-\epsilon}} J(\epsilon-1, 3-3\epsilon, 1) - \frac{(1-2\epsilon^2)(2-3\epsilon)}{[31]^2[12]^{1-\epsilon}} J(1+\epsilon, 1-3\epsilon, 1) \\
& \quad + \left( \frac{\epsilon(2-\epsilon)[21]}{[31][23]^{1-\epsilon}} - \frac{(2-\epsilon)(2-3\epsilon)}{[31][23]^{-\epsilon}} \right) J(2, 2-3\epsilon, 1+\epsilon) \\
& + \left( \frac{(\epsilon^2-2\epsilon)[21]}{[31]^2[23]^{1-\epsilon}} + \frac{4-8\epsilon+3\epsilon^2}{[31]^2[23]^{-\epsilon}} + \frac{\epsilon^2-\epsilon}{[31][23]^{1-\epsilon}} \right) J(1, 2-3\epsilon, 1+\epsilon) \\
& + \left( \frac{4-10\epsilon+6\epsilon^2}{[31]^2[23]^{-\epsilon}} - \frac{\epsilon(1-2\epsilon)}{[31][23]^{1-\epsilon}} + \frac{(3\epsilon-4\epsilon^2)[21]}{[31]^2[23]^{1-\epsilon}} \right) J(1, 3-3\epsilon, \epsilon) \\
& \quad - \frac{(2-\epsilon)(2-3\epsilon)[21]}{[31][23]^{-\epsilon}} J(2, 3-3\epsilon, 1+\epsilon) \\
& + \left( \frac{(2-\epsilon)(2-3\epsilon)[21]}{[31]^2[23]^{-\epsilon}} + \frac{(2-\epsilon)(2-3\epsilon)}{[31][23]^{-\epsilon}} \right) J(1, 3-3\epsilon, 1+\epsilon) \\
& \quad + \left[ \frac{\epsilon(1+\epsilon)[21]}{[31]^2[23]^{1-\epsilon}} + \frac{\epsilon(1-\epsilon)[21]}{[31][23]^{2-\epsilon}} - \frac{[12]^2\epsilon(1-\epsilon)}{[31]^2[23]^{2-\epsilon}} \right. \\
& \quad \left. - \frac{2(1-\epsilon)(2-3\epsilon)}{[31]^2[23]^{-\epsilon}} + \frac{\epsilon(1-2\epsilon)}{[31][23]^{1-\epsilon}} \right] J(2, 2-3\epsilon, \epsilon) \\
& + \left[ -\frac{2\epsilon(1-\epsilon)[21]}{[31]^2[23]^{1-\epsilon}} - \frac{\epsilon(1-\epsilon)[21]}{[31][23]^{2-\epsilon}} + \frac{[12]^2\epsilon(1-\epsilon)}{[31]^2[23]^{2-\epsilon}} \right] J(2, 3-3\epsilon, \epsilon-1) \\
& + \left[ -\frac{2(2-3\epsilon)(1-\epsilon)[21]}{[31]^2[23]^{-\epsilon}} + \frac{\epsilon(1-2\epsilon)[21]}{[31][23]^{1-\epsilon}} - \frac{[12]^2\epsilon(2-3\epsilon)}{[31]^2[23]^{1-\epsilon}} \right] J(2, 3-3\epsilon, \epsilon) \\
& + \frac{1-3\epsilon}{[21]^{-\epsilon}[31]} J(1+\epsilon, 2-3\epsilon, 2) + \left( -\frac{1-3\epsilon}{[31]^2[21]^{-\epsilon}} + \frac{\epsilon}{[31][21]^{1-\epsilon}} \right) J(\epsilon, 2-3\epsilon, 2) \\
& \quad - \frac{\epsilon}{[31]^2[21]^{1-\epsilon}} J(\epsilon-1, 2-3\epsilon, 2) \\
& + \frac{2-3\epsilon}{[31][21]^{1-\epsilon}} J(1+\epsilon, 1-3\epsilon, 2) - \frac{2-3\epsilon}{[31]^2[21]^{1-\epsilon}} J(\epsilon, 1-3\epsilon, 2) \\
& \quad + \frac{\epsilon(1-\epsilon)}{[31][23]^{1-\epsilon}} J(2, 1-3\epsilon, 1+\epsilon) \\
& + \left( -\frac{(1-2\epsilon)(1-\epsilon)}{[31]^2[23]^{1-\epsilon}} - \frac{(1-\epsilon)^2}{[31][23]^{2-\epsilon}} + \frac{(1-\epsilon)^2[12]}{[31]^2[23]^{2-\epsilon}} \right) J(2, 1-3\epsilon, \epsilon) \\
& \quad - \frac{\epsilon(1-\epsilon)}{[31]^2[23]^{1-\epsilon}} J(1, 1-3\epsilon, 1+\epsilon)
\end{aligned}$$

$$\begin{aligned}
& + \left( -\frac{2\epsilon(1-\epsilon)}{[31]^2[23]^{1-\epsilon}} - \frac{\epsilon(1-\epsilon)}{[31][23]^{2-\epsilon}} + \frac{\epsilon(1-\epsilon)[12]}{[31]^2[23]^{2-\epsilon}} \right) J(2, 2-3\epsilon, \epsilon-1) \\
& + \left( \frac{(1-\epsilon)^2}{[31]^2[23]^{1-\epsilon}} + \frac{(1-\epsilon)^2}{[31][23]^{2-\epsilon}} - \frac{(1-\epsilon)^2[21]}{[31]^2[23]^{2-\epsilon}} \right) J(1, 2-3\epsilon, \epsilon) \\
& + \frac{\epsilon(1-\epsilon)}{[31]^2[23]^{1-\epsilon}} J(0, 2-3\epsilon, 1+\epsilon) \\
& + \left( \frac{2\epsilon(1-\epsilon)}{[31]^2[23]^{1-\epsilon}} + \frac{\epsilon(1-\epsilon)}{[31][23]^{2-\epsilon}} - \frac{\epsilon(1-\epsilon)[12]}{[31]^2[23]^{2-\epsilon}} \right) J(1, 3-3\epsilon, \epsilon-1) \\
& - \frac{\epsilon(1-\epsilon)}{[31]^2[23]^{1-\epsilon}} J(0, 3-3\epsilon, \epsilon) \\
& + \left( \frac{1-\epsilon}{[31][23]^{2-\epsilon}} + \frac{1-\epsilon}{[31]^2[23]^{1-\epsilon}} - \frac{(1-\epsilon)[12]}{[31]^2[23]^{2-\epsilon}} \right) J(2, 1-2\epsilon, 0) \\
& - \left( \frac{1-\epsilon}{[31][23]^{2-\epsilon}} + \frac{1-\epsilon}{[31]^2[23]^{1-\epsilon}} - \frac{(1-\epsilon)[12]}{[31]^2[23]^{2-\epsilon}} \right) J(1, 2-2\epsilon, 0) \Big\} . \quad (\text{A.1})
\end{aligned}$$

The third double integral (6) is simpler, the integration over  $y$  can be taken immediately,

$$\begin{aligned}
I_{b,3} &= \frac{A(1, 1-2\epsilon, 1)}{4\epsilon(1-2\epsilon)} \left\{ -\frac{1-2\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(1+\epsilon, 1-3\epsilon, 1) \right. \\
& \left( -\frac{1+\epsilon-4\epsilon^2}{[12]^{-\epsilon}[31]^2} + \frac{(2\epsilon-2\epsilon^2)[23]}{[12]^{1-\epsilon}[31]^2} + \frac{1-2\epsilon-2\epsilon^2}{[12]^{1-\epsilon}[31]} \right) J(1+\epsilon, 2-3\epsilon, 1) \\
& + \frac{1+2\epsilon-4\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(\epsilon, 2-3\epsilon, 1) - \frac{2+2\epsilon-4\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(1+\epsilon, 2-3\epsilon, 0) \\
& - \frac{2\epsilon-2\epsilon^2}{[12]^{-\epsilon}[31]^2} J(1+\epsilon, 3-3\epsilon, 0) + \frac{2\epsilon-2\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(\epsilon, 3-3\epsilon, 0) \\
& + \left( \frac{(2\epsilon-2\epsilon^2)[23]}{[12]^{-\epsilon}[31]^2} - \frac{\epsilon}{[12]^{-\epsilon}[31]} - \frac{\epsilon-2\epsilon^2}{[12]^{-\epsilon-1}[31]^2} \right) J(1+\epsilon, 3-3\epsilon, 1) \\
& + \frac{2+\epsilon}{[12]^{1-\epsilon}[31]} J(1+\epsilon, 1-3\epsilon, 2) \\
& + \left( -\frac{2+\epsilon}{[12]^{1-\epsilon}[31]} - \frac{\epsilon[23]}{[12]^{1-\epsilon}[31]^2} - \frac{1}{[12]^{-\epsilon}[31]^2} \right) J(\epsilon, 2-3\epsilon, 2) \\
& + \left( -\frac{2\epsilon(1-\epsilon)[23]}{[12]^{1-\epsilon}[31]^2} + \frac{\epsilon}{[12]^{1-\epsilon}[31]} + \frac{3\epsilon-4\epsilon^2}{[12]^{-\epsilon}[31]^2} \right) J(\epsilon, 3-3\epsilon, 1) \\
& + \frac{\epsilon[23]}{[12]^{-\epsilon}[31]} J(1+\epsilon, 3-3\epsilon, 2) - \left( \frac{\epsilon[23]}{[12]^{1-\epsilon}[31]} + \frac{\epsilon[23]}{[12]^{-\epsilon}[31]^2} \right) J(\epsilon, 3-3\epsilon, 2) \\
& - \frac{1}{[12]^{1-\epsilon}[31]^2} J(\epsilon, 1-3\epsilon, 2) + \frac{1}{[12]^{1-\epsilon}[31]^2} J(\epsilon-1, 2-3\epsilon, 2) \\
& - \frac{2\epsilon-2\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(\epsilon-1, 3-3\epsilon, 1) + \frac{\epsilon[23]}{[12]^{1-\epsilon}[31]^2} J(\epsilon-1, 3-3\epsilon, 2)
\end{aligned}$$

$$\begin{aligned}
& + \frac{2 - 2\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(2 + \epsilon, 1 - 3\epsilon, 0) \\
& - \frac{1 + \epsilon}{[12]^{1-\epsilon}} J(2 + \epsilon, 1 - 3\epsilon, 2) - \frac{1 + \epsilon}{[12]^{-\epsilon}} J(2 + \epsilon, 2 - 3\epsilon, 2) \\
& + \left( \frac{1 + \epsilon}{[12]^{1-\epsilon}} + \frac{2 + \epsilon}{[12]^{-\epsilon}[31]} + \frac{\epsilon[23]}{[12]^{1-\epsilon}[31]} \right) J(1 + \epsilon, 2 - 3\epsilon, 2) \\
& - \frac{1 - \epsilon - 2\epsilon^2}{[12]^{1-\epsilon}[31]} J(2 + \epsilon, 1 - 3\epsilon, 1) - \frac{1 - \epsilon - 2\epsilon^2}{[12]^{-\epsilon}[31]} J(2 + \epsilon, 2 - 3\epsilon, 1) \Big\} . \tag{A.2}
\end{aligned}$$

In Eqs. (A.1) and (A.2), we do not write single integrals with one of the indices zero and sum of other two indices equal to  $D$ ,  $D + 1$  or  $D + 2$ . Such terms are proportional to delta-functions in position space multiplied by pole in  $\epsilon$  and will disappear in the final expression (13).

## Appendix B

In this Appendix we write the results for the terms contributing to the diagram (b) with  $J$ 's whose sum of indices is  $k = D, D + 1, D + 2$ . These terms appear in  $I_b$  which is the sum of Eqs. (7), (A.1) and (A.2). The integrals in  $I_b^{(D)}$  are calculated by using the uniqueness method

$$\begin{aligned}
I_b^{(D)} = & \frac{A(1, 1 - 2\epsilon, 1)}{4\epsilon(1 + 2\epsilon)(1 - 2\epsilon)^2} \Big\{ \left( \frac{10 - 18\epsilon - 12\epsilon^2}{[31]^2[12]^{-\epsilon}} + \frac{3 - 4\epsilon - 10\epsilon^2}{[31][12]^{1-\epsilon}} + \frac{(2\epsilon - 2\epsilon^2)[23]}{[31]^2[12]^{1-\epsilon}} \right) J(1 + \epsilon, 2 - 3\epsilon, 1) \\
& + \frac{-3 + 3\epsilon + 12\epsilon^2}{[31][12]^{1-\epsilon}} J(2 + \epsilon, 1 - 3\epsilon, 1) + \frac{2\epsilon - 4\epsilon^2}{[31]^2[12]^{1-\epsilon}} J(2 + \epsilon, 1 - 3\epsilon, 0) \\
& + \frac{-4\epsilon + 8\epsilon^2}{[31]^2[12]^{1-\epsilon}} J(1 + \epsilon, 2 - 3\epsilon, 0) + \frac{-2\epsilon + 2\epsilon^2}{[31][23]^{1-\epsilon}} J(2, 1 - 3\epsilon, 1 + \epsilon) \\
& + \left[ \frac{-8 + 16\epsilon - 6\epsilon^2}{[31]^2[23]^{-\epsilon}} + \frac{-\epsilon + 3\epsilon^2}{[31][23]^{1-\epsilon}} + \frac{(-2\epsilon + 5\epsilon^2)[21]}{[31]^2[23]^{1-\epsilon}} \right] J(1, 2 - 3\epsilon, 1 + \epsilon) \\
& + \left[ \frac{(7\epsilon - 14\epsilon^2)[21]}{[31]^2[23]^{1-\epsilon}} + \frac{(\epsilon - 3\epsilon^2)[21]}{[31][23]^{2-\epsilon}} + \frac{[12]^2(-\epsilon + 3\epsilon^2)}{[31]^2[23]^{2-\epsilon}} \right. \\
& \quad \left. + \frac{8 - 20\epsilon + 12\epsilon^2}{[31]^2[23]^{-\epsilon}} + \frac{-2\epsilon + 4\epsilon^2}{[31][23]^{1-\epsilon}} \right] J(2, 2 - 3\epsilon, \epsilon) \\
& + \frac{1 + 2\epsilon - 6\epsilon^2}{[12]^{1-\epsilon}[31]} J(1 + \epsilon, 1 - 3\epsilon, 2) + \left( \frac{-2 + 2\epsilon}{[31][23]^{2-\epsilon}} + \frac{-2 + 2\epsilon}{[31]^2[23]^{1-\epsilon}} + \frac{[12](2 - 2\epsilon)}{[31]^2[23]^{2-\epsilon}} \right) J(2, 1 - 2\epsilon, 0) \\
& + \frac{2\epsilon - 4\epsilon^2}{[12]^{1-\epsilon}[31]^2} J(\epsilon, 3 - 3\epsilon, 0) + \left( \frac{-2 + 6\epsilon^2}{[12]^{1-\epsilon}[31]} + \frac{(-\epsilon)[23]}{[12]^{1-\epsilon}[31]^2} + \frac{-2 + 5\epsilon - 2\epsilon^2}{[12]^{-\epsilon}[31]^2} \right) J(\epsilon, 2 - 3\epsilon, 2) \\
& + \left( \frac{(-2\epsilon + 2\epsilon^2)[23]}{[12]^{1-\epsilon}[31]^2} + \frac{\epsilon - 2\epsilon^2}{[12]^{1-\epsilon}[31]} + \frac{4 - 11\epsilon + 2\epsilon^2}{[12]^{-\epsilon}[31]^2} \right) J(\epsilon, 3 - 3\epsilon, 1) \\
& + \left( \frac{4 - 18\epsilon + 26\epsilon^2}{[31]^2[23]^{-\epsilon}} + \frac{-\epsilon + 4\epsilon^2}{[31][23]^{1-\epsilon}} + \frac{(3\epsilon - 10\epsilon^2)[21]}{[31]^2[23]^{1-\epsilon}} \right) J(1, 3 - 3\epsilon, \epsilon)
\end{aligned}$$

$$\begin{aligned}
& + \frac{(\epsilon)[23]}{[12]^{1-\epsilon}[31]^2} J(\epsilon-1, 3-3\epsilon, 2) \\
& + \left[ \frac{(-2\epsilon+6\epsilon^2)[21]}{[31]^2[23]^{1-\epsilon}} + \frac{(-\epsilon+3\epsilon^2)[21]}{[31][23]^{2-\epsilon}} + \frac{[12]^2(\epsilon-3\epsilon^2)}{[31]^2[23]^{2-\epsilon}} \right] J(2, 3-3\epsilon, \epsilon-1) \\
& + \frac{\epsilon-3\epsilon^2}{[31]^2[23]^{1-\epsilon}} J(0, 2-3\epsilon, 1+\epsilon) + \frac{-\epsilon+3\epsilon^2}{[31]^2[23]^{1-\epsilon}} J(0, 3-3\epsilon, \epsilon) \\
& + \left( \frac{-1+3\epsilon-2\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{-1+3\epsilon-2\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{(1-3\epsilon+2\epsilon^2)[12]}{[31]^2[23]^{2-\epsilon}} \right) J(1, 2-2\epsilon, 0) \Big\} \\
& = \frac{A(1, 1-2\epsilon, 1)A(1+\epsilon, 1-3\epsilon, 1)}{8\epsilon^2(1+2\epsilon)(1-2\epsilon)^2} \left\{ \frac{-2\epsilon+10\epsilon^2}{[12]^{1-2\epsilon}[23]^{1-2\epsilon}[31]^{2+2\epsilon}} \right. \\
& + \frac{3+7\epsilon+3\epsilon^2}{[12]^{2-2\epsilon}[23]^{1-2\epsilon}[13]^{1+2\epsilon}} + \frac{-5+8\epsilon-18\epsilon^2}{[12]^{2-2\epsilon}[23]^{-2\epsilon}[13]^{2+2\epsilon}} + \frac{-6\epsilon+\epsilon^2}{[31]^2[12]^{2-2\epsilon}} \\
& + \frac{2-4\epsilon-5\epsilon^2}{[12]^{1-2\epsilon}[23]^{2-2\epsilon}[31]^{1+2\epsilon}} + \frac{2-4\epsilon}{[12]^{-2\epsilon}[23]^{2-2\epsilon}[31]^{2+2\epsilon}} + \frac{\epsilon-(1/2)\epsilon^2}{[12]^{2-2\epsilon}[23]^{2-2\epsilon}[31]^{2\epsilon}} + \frac{\epsilon+\epsilon^2/2}{[31]^2[23]^{2-2\epsilon}} \Big\} \\
& + \frac{A^2(1, 1-2\epsilon, 1)}{4\epsilon^2(1+2\epsilon)(1-2\epsilon)^2} \left\{ \frac{1-\epsilon}{[12]^{1-\epsilon}[23]^{2-\epsilon}[31]} + \frac{1-\epsilon}{[12]^{1-\epsilon}[23]^{1-\epsilon}[31]^2} - \frac{1-\epsilon}{[12]^{-\epsilon}[23]^{2-\epsilon}[31]^2} \right\} \quad (B.1)
\end{aligned}$$

The terms in the sums  $I_b^{(D+1)}$  and  $I_b^{(D+2)}$  can be calculated also by using the uniqueness method, but only after having represented the  $J$ -terms as derivatives of integrals  $J(\alpha_1, \alpha_2, \alpha_3)$  with  $\sum \alpha_j = D$ .

$$\begin{aligned}
I_b^{(D+1)} + I_b^{(D+2)} &= \frac{A(1, 1-2\epsilon, 1)}{4\epsilon(1+2\epsilon)(1-2\epsilon)^2} \left\{ \frac{-11+13\epsilon+32\epsilon^2}{[31][12]^{-\epsilon}} J(2+\epsilon, 2-3\epsilon, 1) \right. \\
& + \left[ \frac{8-16\epsilon+6\epsilon^2}{[31][23]^{-\epsilon}} + \frac{(2\epsilon-5\epsilon^2)[21]}{[31][23]^{1-\epsilon}} \right] J(2, 2-3\epsilon, 1+\epsilon) \\
& + \left( \frac{(2\epsilon-2\epsilon^2)[23]}{[12]^{-\epsilon}[31]^2} + \frac{-4+13\epsilon-6\epsilon^2}{[12]^{-\epsilon}[31]} + \frac{-4+13\epsilon-6\epsilon^2}{[12]^{-\epsilon-1}[31]^2} \right) J(1+\epsilon, 3-3\epsilon, 1) \\
& + \frac{(\epsilon)[23]}{[12]^{-\epsilon}[31]} J(1+\epsilon, 3-3\epsilon, 2) - \left( \frac{(\epsilon)[23]}{[12]^{1-\epsilon}[31]} + \frac{(\epsilon)[23]}{[12]^{-\epsilon}[31]^2} \right) J(\epsilon, 3-3\epsilon, 2) \\
& + \frac{-1-\epsilon+4\epsilon^2}{[12]^{1-\epsilon}} J(2+\epsilon, 1-3\epsilon, 2) + \frac{-1-\epsilon+4\epsilon^2}{[12]^{-\epsilon}} J(2+\epsilon, 2-3\epsilon, 2) \\
& + \left( \frac{1+\epsilon-4\epsilon^2}{[12]^{1-\epsilon}} + \frac{3-4\epsilon-2\epsilon^2}{[12]^{-\epsilon}[31]} + \frac{(\epsilon)[23]}{[12]^{1-\epsilon}[31]} \right) J(1+\epsilon, 2-3\epsilon, 2) \\
& + \frac{4-14\epsilon+8\epsilon^2}{[31][12]^{-1-\epsilon}} J(2+\epsilon, 3-3\epsilon, 1) \\
& + \frac{(-4+16\epsilon-19\epsilon^2)[21]}{[31][23]^{-\epsilon}} J(2, 3-3\epsilon, 1+\epsilon) \\
& + \left( \frac{(4-16\epsilon+19\epsilon^2)[21]}{[31]^2[23]^{-\epsilon}} + \frac{4-16\epsilon+19\epsilon^2}{[31][23]^{-\epsilon}} \right) J(1, 3-3\epsilon, 1+\epsilon)
\end{aligned}$$

$$\begin{aligned}
& + \left[ \frac{(-4 + 18\epsilon - 26\epsilon^2)[21]}{[31]^2[23]^{-\epsilon}} + \frac{(\epsilon - 4\epsilon^2)[21]}{[31][23]^{1-\epsilon}} + \frac{[12]^2(-2\epsilon + 7\epsilon^2)}{[31]^2[23]^{1-\epsilon}} \right] J(2, 3 - 3\epsilon, \epsilon) \Big\} \\
& = \frac{A(1, 1 - 2\epsilon, 1)A(1 - 3\epsilon, 1, 1 + \epsilon)}{8\epsilon^2(1 + 2\epsilon)(1 - 2\epsilon)^2} \left\{ \frac{2 - (17/2)\epsilon - (35/4)\epsilon^2}{[12]^{1-2\epsilon}[23]^{1-2\epsilon}[13]^{2+2\epsilon}} \right. \\
& \quad + \frac{-3 - (3/2)\epsilon - (7/4)\epsilon^2}{[12]^{2-2\epsilon}[23]^{1-2\epsilon}[13]^{1+2\epsilon}} + \frac{5 - 5\epsilon + (39/2)\epsilon^2}{[12]^{2-2\epsilon}[23]^{-2\epsilon}[13]^{2+2\epsilon}} \\
& \quad + \frac{1 - 4\epsilon + (21/2)\epsilon^2}{[12]^{1-2\epsilon}[23]^{2-2\epsilon}[31]^{1+2\epsilon}} + \frac{-4 + (15/2)\epsilon - (19/4)\epsilon^2}{[12]^{-2\epsilon}[23]^{2-2\epsilon}[31]^{2+2\epsilon}} + \left. \frac{(5/2)\epsilon - (7/4)\epsilon^2}{[12]^{2-2\epsilon}[23]^{2-2\epsilon}[31]^{2\epsilon}} \right\}. \quad (\text{B.2})
\end{aligned}$$

## Appendix C

In this Appendix we provide formulas used in Sec. 4. The following formula results from applying the corresponding IBP procedure:

$$\begin{aligned}
J(1, 3 - 3\epsilon, \epsilon - 1) &= -\frac{1}{2 - 3\epsilon} [J(2, 2 - 3\epsilon, \epsilon - 1) - [12]J(2, 3 - 3\epsilon, \epsilon - 1) \\
&\quad + (\epsilon - 1)(J(1, 2 - 3\epsilon, \epsilon) - [23]J(1, 3 - 3\epsilon, \epsilon))] = \\
&= -\frac{1}{2 - 3\epsilon} \left[ J(2, 2 - 3\epsilon, \epsilon - 1) - \frac{A(2, 3 - 3\epsilon, \epsilon - 1)}{[12]^{2-2\epsilon}[23]^{-\epsilon}[31]^{-1+2\epsilon}} \right. \\
&\quad \left. + (\epsilon - 1) \left( J(1, 2 - 3\epsilon, \epsilon) - \frac{A(1, 3 - 3\epsilon, \epsilon)}{[12]^{2-2\epsilon}[23]^{-\epsilon}[31]^{-1+2\epsilon}} \right) \right]. \quad (\text{C.1})
\end{aligned}$$

Other useful formulas can be taken from Ref. [9] (Appendix A there, obtained by IBP procedure):

$$\begin{aligned}
J(1 - 3\epsilon, 2, \epsilon) &= -(1 - 3\epsilon)J(2 - 3\epsilon, 1, \epsilon) - \epsilon J(1 - 3\epsilon, 1, 1 + \epsilon) \\
&\quad - \frac{A(1, 1 - 3\epsilon, 1 + \epsilon)}{2\epsilon} \frac{1}{[12]^{1-2\epsilon}[13]^{-\epsilon}[23]^{2\epsilon}}, \quad (\text{C.2})
\end{aligned}$$

$$\begin{aligned}
J(2, 1 - 3\epsilon, \epsilon) &= -(1 - 3\epsilon)J(1, 2 - 3\epsilon, \epsilon) - \epsilon J(1, 1 - 3\epsilon, 1 + \epsilon) \\
&\quad - \frac{A(1, 1 - 3\epsilon, 1 + \epsilon)}{2\epsilon} \frac{1}{[12]^{1-2\epsilon}[23]^{-\epsilon}[31]^{2\epsilon}}, \quad (\text{C.3})
\end{aligned}$$

$$\begin{aligned}
J(\epsilon, 1 - 3\epsilon, 2) &= -(1 - 3\epsilon)J(\epsilon, 2 - 3\epsilon, 1) - \epsilon J(1 + \epsilon, 1 - 3\epsilon, 1) \\
&\quad - \frac{A(1, 1 - 3\epsilon, 1 + \epsilon)}{2\epsilon} \frac{1}{[23]^{1-2\epsilon}[12]^{-\epsilon}[31]^{2\epsilon}}. \quad (\text{C.4})
\end{aligned}$$

The following useful relation can be directly derived:

$$\begin{aligned}
J(2, 2 - 3\epsilon, \epsilon - 1) &\equiv \int Dx \frac{1}{[x1]^2[x2]^{2-3\epsilon}[x3]^{\epsilon-1}} = \int Dx \frac{[x3]}{[x1]^2[x2]^{2-3\epsilon}[x3]^{\epsilon}} \\
&= \int Dx \frac{[x1] + [13] + 2(x1)(13)}{[x1]^2[x2]^{2-3\epsilon}[x3]^{\epsilon}} \\
&= J(1, 2 - 3\epsilon, \epsilon) + [31]J(2, 2 - 3\epsilon, \epsilon) + 2(13) \int Dx \frac{(x1)}{[x1]^2[x2]^{2-3\epsilon}[x3]^{\epsilon}} \\
&= J(1, 2 - 3\epsilon, \epsilon) + [31]J(2, 2 - 3\epsilon, \epsilon) + (13)_\mu \partial_\mu^{(1)} J(1, 2 - 3\epsilon, \epsilon)
\end{aligned}$$



$$\begin{aligned}
&= J(1, 2 - 3\epsilon, \epsilon) + \frac{A(2, 2 - 3\epsilon, \epsilon)}{[12]^{2-2\epsilon}[23]^{-\epsilon}[31]^{-1+2\epsilon}} + (13)_\mu \partial_\mu^{(1)} J(1, 2 - 3\epsilon, \epsilon) \\
&= J(1, 2 - 3\epsilon, \epsilon) - \frac{(1 - 2\epsilon)}{2\epsilon(1 - 3\epsilon)} \frac{A(1, 1 - 3\epsilon, 1 + \epsilon)}{[12]^{2-2\epsilon}[23]^{-\epsilon}[31]^{-1+2\epsilon}} + (13)_\mu \partial_\mu^{(1)} J(1, 2 - 3\epsilon, \epsilon), \\
&\quad J(\epsilon - 1, 2 - 3\epsilon, 2) = \\
&= J(\epsilon, 2 - 3\epsilon, 1) - \frac{(1 - 2\epsilon)}{2\epsilon(1 - 3\epsilon)} \frac{A(1, 1 - 3\epsilon, 1 + \epsilon)}{[23]^{2-2\epsilon}[12]^{-\epsilon}[31]^{-1+2\epsilon}} + (31)_\mu \partial_\mu^{(3)} J(\epsilon, 2 - 3\epsilon, 1). \tag{C.5}
\end{aligned}$$

For  $J(2 - 3\epsilon, 1, \epsilon)$  we use specific formulas of Ref. [9] which were obtained by applying the GPT:

$$\begin{aligned}
J(2 - 3\epsilon, 1, \epsilon) &= \frac{1}{[12]^{1-\epsilon}} A(1 - 3\epsilon, 1, 1 + \epsilon) \frac{1}{2(1 - 3\epsilon)} \times \\
&\left[ \frac{1}{\epsilon} - \ln \frac{[13]}{[12]} - \epsilon \left( \frac{1}{2} \ln \frac{[12][13]}{[23]^2} \ln \frac{[13]}{[12]} + ([12] + [23] - [13]) J(1, 1, 1) \right) \right], \\
J(1, 2 - 3\epsilon, \epsilon) &= \frac{1}{[12]^{1-\epsilon}} A(1 - 3\epsilon, 1, 1 + \epsilon) \frac{1}{2(1 - 3\epsilon)} \times \\
&\left[ \frac{1}{\epsilon} - \ln \frac{[23]}{[12]} - \epsilon \left( \frac{1}{2} \ln \frac{[12][23]}{[13]^2} \ln \frac{[23]}{[12]} + ([12] + [13] - [23]) J(1, 1, 1) \right) \right], \\
J(\epsilon, 2 - 3\epsilon, 1) &= \frac{1}{[23]^{1-\epsilon}} A(1 - 3\epsilon, 1, 1 + \epsilon) \frac{1}{2(1 - 3\epsilon)} \times \\
&\left[ \frac{1}{\epsilon} - \ln \frac{[12]}{[23]} - \epsilon \left( \frac{1}{2} \ln \frac{[23][12]}{[13]^2} \ln \frac{[12]}{[23]} + ([23] + [13] - [12]) J(1, 1, 1) \right) \right]. \tag{C.6}
\end{aligned}$$

## Appendix D

In this appendix we apply formulas (C.6) and obtain for the term in Eq. (11) proportional to  $\partial_\mu^{(1)} J(1, 2 - 3\epsilon, \epsilon)$

$$\begin{aligned}
I_b^{(\partial J, 1)} &\equiv \frac{A(1, 1 - 2\epsilon, 1)}{4\epsilon(1 + 2\epsilon)(1 - 2\epsilon)^2} \left[ \frac{3\epsilon - (5/2)\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{(3/2)\epsilon - (5/4)\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{[12](-(3/2)\epsilon + (5/4)\epsilon^2)}{[31]^2[23]^{2-\epsilon}} \right] \times \\
&\quad \times (13)_\mu \partial_\mu^{(1)} J(1, 2 - 3\epsilon, \epsilon) = \\
&= -\frac{A(1, 1 - 2\epsilon, 1)A(1 - 3\epsilon, 1, 1 + \epsilon)}{8\epsilon(1 + 2\epsilon)(1 - 2\epsilon)^2} \left[ \frac{9/2}{[12]^{1-\epsilon}[31]^2[23]^{1-\epsilon}} + \frac{3/2}{[12]^{2-\epsilon}[31][23]^{1-\epsilon}} \right. \\
&\quad \left. + \frac{-3}{[12]^{2-\epsilon}[31]^2[23]^{-\epsilon}} + \frac{3/2}{[12]^{2-\epsilon}[23]^{2-\epsilon}} + \frac{-3/2}{[12]^{-\epsilon}[31]^2[23]^{2-\epsilon}} \right] \\
&\quad - \frac{1}{8} \left[ \frac{21/4}{[12][23][31]^2} + \frac{7/4}{[12]^2[23][31]} + \frac{-7/2}{[12]^2[31]^2} + \frac{7/4}{[12]^2[23]^2} + \frac{-7/4}{[31]^2[23]^2} \right] \\
&\quad - \frac{3}{16} \left[ \frac{2}{[31]^2[23]} + \frac{1}{[31][23]^2} - \frac{[12]}{[31]^2[23]^2} \right] (13)_\mu \partial_\mu^{(1)} \frac{1}{[12]} \ln \frac{[23]}{[12]} \tag{D.1}
\end{aligned}$$

and for the term in Eq. (11) proportional to  $\partial_\mu^{(3)} J(\epsilon, 2 - 3\epsilon, 1)$

$$I_b^{(\partial J, 2)} \equiv \frac{A(1, 1 - 2\epsilon, 1)}{4\epsilon(1 + 2\epsilon)(1 - 2\epsilon)^2} \frac{(1 - (5/2)\epsilon^2)}{[12]^{1-\epsilon}[31]^2} (31)_\mu \partial_\mu^{(3)} J(\epsilon, 2 - 3\epsilon, 1)$$

$$\begin{aligned}
&= \frac{A(1, 1-2\epsilon, 1)}{4\epsilon(1+2\epsilon)(1-2\epsilon)^2} \frac{(1-(5/2)\epsilon^2)}{[12]^{1-\epsilon}[31]^2} (31)_\mu \partial_\mu^{(3)} \left\{ \frac{1}{[23]^{1-\epsilon}} A(1-3\epsilon, 1, 1+\epsilon) \frac{1}{2(1-3\epsilon)} \times \right. \\
&\quad \left. \left[ \frac{1}{\epsilon} - \ln \frac{[12]}{[23]} - \epsilon \left( \frac{1}{2} \ln \frac{[23][12]}{[13]^2} \ln \frac{[12]}{[23]} + ([23] + [13] - [12]) J(1, 1, 1) \right) \right] \right\} \\
&= -\frac{A(1, 1-2\epsilon, 1)A(1-3\epsilon, 1, 1+\epsilon)}{8\epsilon^2(1+2\epsilon)(1-2\epsilon)^2} \left[ \frac{1}{[12]^{1-\epsilon}[23]^{2-\epsilon}[31]} \right. \\
&\quad \left. + \frac{1}{[12]^{1-\epsilon}[23]^{1-\epsilon}[31]^2} - \frac{1}{[12]^{-\epsilon}[23]^{2-\epsilon}[31]^2} \right] \\
&\quad + \frac{A(1, 1-2\epsilon, 1)A(1-3\epsilon, 1, 1+\epsilon)}{8\epsilon(1+2\epsilon)(1-2\epsilon)^2} \left[ \frac{1}{[12]^{1-\epsilon}[23]^{2-\epsilon}[31]} \right. \\
&\quad \left. + \frac{1}{[12]^{1-\epsilon}[23]^{1-\epsilon}[31]^2} - \frac{1}{[12]^{-\epsilon}[23]^{2-\epsilon}[31]^2} \right] \ln \frac{[12]}{[23]} \\
&\quad - \frac{A(1, 1-2\epsilon, 1)A(1-3\epsilon, 1, 1+\epsilon)}{8\epsilon(1+2\epsilon)(1-2\epsilon)^2} \left[ \frac{1}{[12]^{1-\epsilon}[23]^{2-\epsilon}[31]} \right. \\
&\quad \left. + \frac{1}{[12]^{1-\epsilon}[23]^{1-\epsilon}[31]^2} - \frac{1}{[12]^{-\epsilon}[23]^{2-\epsilon}[31]^2} \right] \\
&\quad - \frac{1}{8} \frac{1}{[12][31]^2} (31)_\mu \partial_\mu^{(3)} \frac{1}{[23]} \left( \frac{1}{2} \ln \frac{[23][12]}{[13]^2} \ln \frac{[12]}{[23]} + ([23] + [13] - [12]) J(1, 1, 1) \right) \\
&\quad - \frac{1}{16} \left[ \frac{1}{[12][23]^2[31]} + \frac{1}{[12][23][31]^2} - \frac{1}{[23]^2[31]^2} \right] \\
&\quad + \frac{1}{4} \left[ \frac{1}{[12][23]^2[31]} + \frac{1}{[12][23][31]^2} - \frac{1}{[23]^2[31]^2} \right] \ln \frac{[12]}{[23]}. \quad (D.2)
\end{aligned}$$

## Appendix E

Collecting (12), (D.1) and (D.2) in Eq. (11), we obtain the final expression for the diagram (b) in  $D$  dimensions

$$\begin{aligned}
I_b &= \frac{A(1, 1-2\epsilon, 1)}{4\epsilon(1+2\epsilon)(1-2\epsilon)^2} \left\{ \frac{5\epsilon - 12\epsilon^2}{[31]^2[12]^{1-\epsilon}} J(\epsilon, 2-3\epsilon, 1) \right. \\
&\quad + \left[ \frac{-1+9\epsilon-17\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{-1+8\epsilon-(23/2)\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{(1-8\epsilon+(23/2)\epsilon^2)[21]}{[31]^2[23]^{2-\epsilon}} \right] J(1, 2-3\epsilon, \epsilon) \\
&\quad + \left[ \frac{3\epsilon-(5/2)\epsilon^2}{[31]^2[23]^{1-\epsilon}} + \frac{-(3/2)\epsilon+(5/4)\epsilon^2}{[31][23]^{2-\epsilon}} + \frac{[12](-(3/2)\epsilon+(5/4)\epsilon^2)}{[31]^2[23]^{2-\epsilon}} \right] (13)_\mu \partial_\mu^{(1)} J(1, 2-3\epsilon, \epsilon) \\
&\quad \left. + \frac{1-(5/2)\epsilon^2}{[12]^{1-\epsilon}[31]^2} (31)_\mu \partial_\mu^{(3)} J(\epsilon, 2-3\epsilon, 1) \right\} \\
&\quad - \frac{1}{2} \left[ \frac{1}{[31]^2[23]} + \frac{1}{[31][23]^2} - \frac{[12]}{[31]^2[23]^2} \right] J(1, 1, 1) \\
&\quad + \frac{1}{8} \left\{ \frac{-27/2}{[12][23][31]^2} + \frac{5}{[12][23]^2[31]} + \frac{1/4}{[12]^2[23][31]} + \frac{-11/4}{[12]^2[23]^2} + \frac{-9/4}{[23]^2[31]^2} + \frac{5/2}{[12]^2[31]^2} \right\}
\end{aligned}$$

$$\begin{aligned}
& + \frac{A(1, 1 - 2\epsilon, 1)A(1, 1 - 3\epsilon, 1 + \epsilon)}{8\epsilon(1 + 2\epsilon)(1 - 2\epsilon)^2} \left\{ \frac{-7/2}{[12]^{1-2\epsilon}[23]^{1-2\epsilon}[31]^{2+2\epsilon}} + \frac{-2}{[12]^{1-2\epsilon}[23]^{2-2\epsilon}[31]^{1+2\epsilon}} \right. \\
& \quad + \frac{-1/2}{[12]^{-2\epsilon}[23]^{2-2\epsilon}[31]^{2+2\epsilon}} + \frac{3/2}{[12]^{2-2\epsilon}[23]^{1-2\epsilon}[31]^{1+2\epsilon}} + \frac{3/2}{[12]^{2-2\epsilon}[23]^{2-2\epsilon}[31]^{2\epsilon}} \\
& \quad \left. + \frac{3}{[12]^{2-2\epsilon}[23]^{-2\epsilon}[31]^{2+2\epsilon}} + \frac{-6}{[31]^2[12]^{2-2\epsilon}} + \frac{1}{[31]^2[23]^{2-2\epsilon}} \right\} \\
& + \frac{A^2(1, 1 - 2\epsilon, 1)}{4\epsilon^2(1 + 2\epsilon)(1 - 2\epsilon)^2} \left\{ \frac{1 - \epsilon}{[12]^{1-\epsilon}[23]^{2-\epsilon}[31]} + \frac{1 - \epsilon}{[12]^{1-\epsilon}[23]^{1-\epsilon}[31]^2} - \frac{1 - \epsilon}{[12]^{-\epsilon}[23]^{2-\epsilon}[31]^2} \right\} .(E.1)
\end{aligned}$$

Taking the limit  $\epsilon \rightarrow 0$ , the poles in  $\epsilon$  disappear and the result is

$$\begin{aligned}
I_b(\epsilon \rightarrow 0) = & \frac{1}{8} \left[ \frac{7/2}{[12][23][31]^2} \ln \frac{[31]^2}{[12][23]} + \frac{2}{[12][23]^2[31]} \ln \frac{[31]^2}{[12][23]} \right. \\
& + \frac{-3/2}{[12]^2[23][31]} \ln \frac{[31]^2}{[12][23]} + \frac{-1/2}{[23]^2[31]^2} \ln \frac{[12]^3}{[23][31]^2} \\
& \left. + \frac{3}{[12]^2[31]^2} \ln \frac{[23]^3}{[12][31]^2} + \frac{-3/2}{[12]^2[23]^2} \ln \frac{[31]^2}{[12][23]} \right] \\
& + \frac{1}{16} \left[ \frac{1}{[12][31]^2[23]} + \frac{1}{[12][31][23]^2} + \frac{-1}{[31]^2[23]^2} \right] \ln \frac{[12][23]}{[13]^2} \ln \frac{[23]}{[12]} \\
& + \frac{1}{8} \left[ \frac{-2}{[31]^2[23]} + \frac{3[12]}{[31]^2[23]^2} + \frac{1}{[12][23]^2} - \frac{1}{[31]^2[12]} + \frac{-4}{[31][23]^2} \right] J(1, 1, 1) \\
& - \frac{3}{16} \left[ \frac{2}{[31]^2[23]} + \frac{1}{[31][23]^2} - \frac{[12]}{[31]^2[23]^2} \right] (13)_\mu \partial_\mu^{(1)} \frac{1}{[12]} \ln \frac{[23]}{[12]} \\
& - \frac{1}{8} \frac{1}{[12][31]^2} (31)_\mu \partial_\mu^{(3)} \frac{1}{[23]} \left( \frac{1}{2} \ln \frac{[23][12]}{[13]^2} \ln \frac{[12]}{[23]} + ([23] + [13] - [12])J(1, 1, 1) \right) \\
& + \frac{1}{8} \left[ \frac{7}{[12][23]^2[31]} + \frac{3}{[12][23][31]^2} + \frac{-7}{[23]^2[31]^2} \right] \ln \frac{[12]}{[23]} \\
& + \frac{1}{16} \left\{ \frac{-13}{[12][23][31]^2} + \frac{16}{[12][23]^2[31]} + \frac{-3}{[12]^2[23][31]} + \frac{-9}{[12]^2[23]^2} + \frac{-7}{[23]^2[31]^2} + \frac{12}{[12]^2[31]^2} \right\} .(E.2)
\end{aligned}$$

## References

- [1] G. Cvetič, I. Kondrashuk and I. Schmidt, “Effective action of dressed mean fields for  $N = 4$  super-Yang-Mills theory,” *Mod. Phys. Lett. A* **21** (2006) 1127 [hep-th/0407251].
- [2] I. Kondrashuk and I. Schmidt, “Finiteness of  $n = 4$  super-Yang-Mills effective action in terms of dressed  $n = 1$  superfields,” hep-th/0411150, to appear in *Mod. Phys. Lett. A*.
- [3] G. Cvetič, I. Kondrashuk and I. Schmidt, “On the effective action of dressed mean fields for  $N = 4$  super-Yang-Mills theory,” in *Symmetry, Integrability and Geometry: Methods and Applications*, SIGMA (2006) 002 [math-ph/0601002].

- [4] I. Kondrashuk, G. Cvetič, and I. Schmidt, “Approach to solve Slavnov-Taylor identities in nonsupersymmetric non-Abelian gauge theories,” Phys. Rev. D **67** (2003) 065006 [hep-ph/0203014].
- [5] G. Cvetič, I. Kondrashuk and I. Schmidt, “QCD effective action with dressing functions: Consistency checks in perturbative regime,” Phys. Rev. D **67** (2003) 065007 [hep-ph/0210185].
- [6] I. Kondrashuk, “The solution to Slavnov-Taylor identities in D4  $N = 1$  SYM,” JHEP **0011**, 034 (2000) [hep-th/0007136].
- [7] I. Kondrashuk, “Renormalizations in softly broken  $N = 1$  theories: Slavnov-Taylor identities,” J. Phys. A **33** (2000) 6399 [hep-th/0002096].
- [8] I. Kondrashuk, “An approach to solve Slavnov-Taylor identity in D4  $N = 1$  supergravity,” Mod. Phys. Lett. A **19** (2004) 1291 [gr-qc/0309075].
- [9] G. Cvetič, I. Kondrashuk, A. Kotikov and I. Schmidt, “Towards the two-loop Lcc vertex in Landau gauge,” hep-th/0604112, to appear in Int. J. Mod. Phys. A.
- [10] C. Becchi, A. Rouet and R. Stora, “Renormalization Of The Abelian Higgs-Kibble Model,” Commun. Math. Phys. **42** (1975) 127.
- [11] I. V. Tyutin, “Gauge Invariance In Field Theory And Statistical Physics In Operator Formalism,” LEBEDEV-75-39 (in Russian), 1975.
- [12] A. A. Slavnov, “Ward Identities In Gauge Theories,” Theor. Math. Phys. **10** (1972) 99 [Teor. Mat. Fiz. **10** (1972) 153].
- [13] J. C. Taylor, “Ward Identities And Charge Renormalization Of The Yang-Mills Field,” Nucl. Phys. B **33** (1971) 436.
- [14] A. A. Slavnov, “Renormalization Of Supersymmetric Gauge Theories. 2. Nonabelian Case,” Nucl. Phys. B **97** (1975) 155.
- [15] L. D. Faddeev and A. A. Slavnov, “Gauge Fields. Introduction To Quantum Theory,” Front. Phys. **50**, 1 (1980) [Front. Phys. **83**, 1 (1990)]; “Introduction to quantum theory of gauge fields”, Moscow, Nauka, (1988).
- [16] B. W. Lee, “Transformation Properties Of Proper Vertices In Gauge Theories,” Phys. Lett. B **46** (1973) 214.
- [17] J. Zinn-Justin, “Renormalization Of Gauge Theories,” SACLAY-D.PH-T-74-88, *Lectures given at Int. Summer Inst. for Theoretical Physics, Jul 29 - Aug 9, 1974, Bonn, West Germany*.
- [18] Z. Bern, M. Czakon, L. J. Dixon, D. A. Kosower and V. A. Smirnov, “The four-loop planar amplitude and cusp anomalous dimension in maximally supersymmetric Yang-Mills theory,” hep-th/0610248.

- [19] Z. Bern, L. J. Dixon and V. A. Smirnov, “Iteration of planar amplitudes in maximally supersymmetric Yang-Mills theory at three loops and beyond,” *Phys. Rev. D* **72** (2005) 085001 [hep-th/0505205].
- [20] A. Blasi, O. Piguet and S. P. Sorella, “Landau gauge and finiteness,” *Nucl. Phys. B* **356** (1991) 154.
- [21] D. Dudal *et al.*, “The anomalous dimension of the gluon-ghost mass operator in Yang-Mills theory,” *Phys. Lett. B* **569** (2003) 57, [hep-th/0306116].
- [22] D. Dudal, H. Verschelde, V. E. R. Lemes, M. S. Sarandy, R. F. Sobreiro, S. P. Sorella and J. A. Gracey, “Renormalizability of the local composite operator  $A(\mu)^{**2}$  in linear covariant gauges,” *Phys. Lett. B* **574** (2003) 325, [hep-th/0308181].
- [23] A. N. Vasiliev, Y. M. Pismak and Y. R. Khonkonen, “1/N Expansion: Calculation Of The Exponents Eta And Nu In The Order  $1/N^{**2}$  For Arbitrary Number Of Dimensions,” *Theor. Math. Phys.* **47** (1981) 465 [*Teor. Mat. Fiz.* **47** (1981) 291].
- [24] D. I. Kazakov, “Analytical Methods For Multiloop Calculations: Two Lectures On The Method Of Uniqueness,” JINR-E2-84-410.
- [25] F. V. Tkachov, “A Theorem On Analytical Calculability Of Four Loop Renormalization Group Functions,” *Phys. Lett. B* **100** (1981) 65.
- [26] K. G. Chetyrkin and F. V. Tkachov, “Integration By Parts: The Algorithm To Calculate Beta Functions In Four Loops,” *Nucl. Phys. B* **192** (1981) 159.
- [27] K. G. Chetyrkin, A. L. Kataev and F. V. Tkachov, “New Approach To Evaluation Of Multiloop Feynman Integrals: The Gegenbauer Polynomial X Space Technique,” *Nucl. Phys. B* **174** (1980) 345.
- [28] W. Celmaster and R. J. Gonsalves, “Fourth Order QCD Contributions To The  $E^+ E^-$  Annihilation Cross-Section,” *Phys. Rev. D* **21** (1980) 3112.
- [29] A. E. Terrano, “A Method For Feynman Diagram Evaluation,” *Phys. Lett. B* **93** (1980) 424.
- [30] B. Lampe and G. Kramer, “Application Of Gegenbauer Integration Method To  $E^+ E^-$  Annihilation Process,” *Phys. Scripta* **28** (1983) 585.
- [31] A. V. Kotikov, “The Gegenbauer Polynomial Technique: the evaluation of a class of Feynman diagrams,” *Phys. Lett. B* **375** (1996) 240 [hep-ph/9512270].
- [32] C. G. Bollini and J. J. Giambiagi, “Dimensional Renormalization: The Number Of Dimensions As A Regularizing Parameter,” *Nuovo Cim. B* **12**, 20 (1972).
- [33] A. I. Davydychev, “Recursive algorithm of evaluating vertex type Feynman integrals,” *J. Phys. A* **25**, 5587 (1992).

- [34] A. V. Kotikov and L. N. Lipatov, “NLO corrections to the BFKL equation in QCD and in supersymmetric gauge theories,” Nucl. Phys. B **582** (2000) 19 [hep-ph/0004008].
- [35] A. V. Kotikov and L. N. Lipatov, “DGLAP and BFKL equations in the  $N = 4$  supersymmetric gauge theory,” Nucl. Phys. B **661** (2003) 19 [Erratum-ibid. B **685** (2004) 405] [hep-ph/0208220].
- [36] A. V. Kotikov, L. N. Lipatov and V. N. Velizhanin, “Anomalous dimensions of Wilson operators in  $N = 4$  SYM theory,” Phys. Lett. B **557** (2003) 114 [hep-ph/0301021].
- [37] A. V. Kotikov, L. N. Lipatov, A. I. Onishchenko and V. N. Velizhanin, “Three-loop universal anomalous dimension of the Wilson operators in  $N = 4$  SUSY Yang-Mills model,” Phys. Lett. B **595** (2004) 521 [Erratum-ibid. B **632** (2006) 754] [hep-th/0404092].
- [38] A. V. Kotikov and L. N. Lipatov, “On the highest transcendentality in  $N = 4$  SUSY,” hep-th/0611204.
- [39] Y. Yamada, “Two loop renormalization group equations for soft SUSY breaking scalar interactions: Supergraph method,” Phys. Rev. D **50** (1994) 3537 [hep-ph/9401241].
- [40] I. Jack and D. R. T. Jones, “The gaugino beta-function,” Phys. Lett. B **415** (1997) 383 [hep-ph/9709364].
- [41] L. V. Avdeev, D. I. Kazakov and I. N. Kondrashuk, “Renormalizations in softly broken SUSY gauge theories,” Nucl. Phys. B **510** (1998) 289 [hep-ph/9709397].
- [42] I. Kondrashuk, “On the relation between Green functions of the SUSY theory with and without soft terms,” Phys. Lett. B **470** (1999) 129 [hep-th/9903167].
- [43] N. E. J. Bjerrum-Bohr, D. C. Dunbar, H. Ita, W. B. Perkins and K. Risager, “The No-triangle hypothesis for  $N = 8$  supergravity,” hep-th/0610043.
- [44] K. Kang and I. Kondrashuk, “Semiclassical scattering amplitudes of dressed gravitons,” hep-ph/0408168.
- [45] M. B. Green, J. G. Russo and P. Vanhove, “Non-renormalisation conditions in type II string theory and maximal supergravity,” hep-th/0610299.
- [46] Z. Bern, L. J. Dixon and R. Roiban, “Is  $N = 8$  supergravity ultraviolet finite?,” Phys. Lett. B **644** (2007) 265 [hep-th/0611086].
- [47] L. V. Avdeev, D. I. Kazakov and I. N. Kondrashuk, “Renormalizations in supersymmetric and nonsupersymmetric nonAbelian Chern-Simons field theories with matter,” Nucl. Phys. B **391** (1993) 333.
- [48] D. R. T. Jones, “Coupling constant renormalization and finite field theory,” Nucl. Phys. B **277** (1986) 153.
- [49] A. V. Ermushev, D. I. Kazakov and O. V. Tarasov, “Finite  $N=1$  Supersymmetric Grand Unified Theories,” Nucl. Phys. B **281** (1987) 72.

- [50] D. I. Kazakov and I. N. Kondrashuk, “Low-Energy Predictions Of Susy Guts: Minimal Versus Finite Model,” *Int. J. Mod. Phys. A* **7** (1992) 3869.
- [51] D. I. Kazakov, M. Y. Kalmykov, I. N. Kondrashuk and A. V. Gladyshev, “Softly Broken Finite Supersymmetric Grand Unified Theory,” *Nucl. Phys. B* **471** (1996) 389 [hep-ph/9511419].
- [52] I. N. Kondrashuk, “Reduction of the finite grand unification theory to the minimal supersymmetric standard model,” *J. Exp. Theor. Phys.* **84** (1997) 432 [*Zh. Eksp. Teor. Fiz.* **111** (1997) 787].
- [53] G. Cvetič, I. Kondrashuk, work in progress.